DEC 31 1928

VOLUME XXIII NEW SERIES, NO. 164

JOURNAL
OF THE
AMERICAN STATISTICAL ASSOCIATION

DECEMBER, 1928

CONTENTS

THE LOGISTIC CURVE AND THE PREDICTION OF THE POPULATION OF THE CHICAGO REGION. By Ardis T. Monk and Helen R. Jeter 361

RECENT IMPROVEMENTS IN GERMAN SOCIAL STATISTICS. By Mollie Ray Carroll 386

ON THE ANALYSIS OF TIME SERIES. By Simon Kuznets 398

DEVICES FOR COMPUTING THE RATES TOGETHER WITH A STATISTICAL STUDY OF BUILDING AND LOAN ASSOCIATIONS. By C. C. Camp 411

FACTORS PRODUCING UNEQUAL WEALTH DISTRIBUTION AND OWNERSHIP AND MEASUREMENTS OF THEIR INFLUENCE. By LeGrand Powers 417

NOTES:

Weekly Wholesale Index. By Irving Fisher 429

The Relationship in a Frequency Polygon of the Average of Any Class to the Mid-Point of the Corresponding Class-Interval. By Willford I. King and Lucile Lawrence 435

Economic and Social Survey of London (440); Freight Traffic of California Cities (440); A Graphic Device for Arithmetic or Geometric Interpolation (441); Progress of the Census (442); Miscellaneous Notes (445); Members Added (449).

REVIEWS 452

RECENT LITERATURE 476

PUBLISHED QUARTERLY BY THE
AMERICAN STATISTICAL ASSOCIATION
PUBLICATION OFFICE: Rumford Press, Concord, N. H.
EDITORIAL OFFICE: Columbia University, New York City

Price $1.50 per copy **$5.00 per annum**

THE EMPLOYMENT CLEARING HOUSE

OF THE

AMERICAN STATISTICAL ASSOCIATION

will aid you with your employment problem. It makes no charge to employers for its services. Any member of the Association may register for a position. There is no fee.

The following positions were open in November, 1928

Field	Pay									
	Under $1800		$1800–2400		$2500–3600		$3700–4900		$5000 and over	
	M	F	M	F	M	F	M	F	M	F
Agricultural Economics					2		1			
Business Forecasting				1						
Business—Mfg. and Selling	1	1	1							
Economic and Industrial Research			4		2		1			
Insurance			1							
Social Work		2				1				

Members were looking for openings as listed:

Field	Pay									
	Under $1800		$1800–2400		$2500–3600		$3700–4900		$5000 and over	
	M	F	M	F	M	F	M	F	M	F
Agricultural Economics					3		2		1	
Business—Mfg. and Selling			2		5		4		3	
Economic and Industrial Research	1				2	1	1		4	
Finance	1	2	1	1	2	1	2		3	
Insurance									1	
Marketing				2	3	2	1			
Social Research		1		1	1	2				1
Teaching					1	1	2		2	
Transportation					1		2			
Vital Statistics						2			1	

COMMUNICATE YOUR NEEDS TO

MRS. ROSE L. WOODBURY

Secretary's Assistant, American Statistical Association

Room 530, Commerce Bldg., 236 Wooster St., New York City

[NOT A PAID ADVERTISEMENT]

NEW SERIES, NO. 164 (VOL. XXIII) DECEMBER, 1928

JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

Formerly the Quarterly Publication of the American Statistical Association

CONTENTS

THE LOGISTIC CURVE AND THE PREDICTION OF THE POPULATION OF THE CHICAGO REGION. By ARDIS T. MONK AND HELEN R. JETER 361

RECENT IMPROVEMENTS IN GERMAN SOCIAL STATISTICS. By MOLLIE RAY CARROLL 386

ON THE ANALYSIS OF TIME SERIES. By SIMON KUZNETS 398

DEVICES FOR COMPUTING THE RATES TOGETHER WITH A STATISTICAL STUDY OF BUILDING AND LOAN ASSOCIATIONS. By C. C. CAMP 411

FACTORS PRODUCING UNEQUAL WEALTH DISTRIBUTION AND OWNERSHIP AND MEASUREMENTS OF THEIR INFLUENCE. By LEGRAND POWERS 417

NOTES:

WEEKLY WHOLESALE INDEX. By IRVING FISHER 429

THE RELATIONSHIP IN A FREQUENCY POLYGON OF THE AVERAGE OF ANY CLASS TO THE MID-POINT OF THE CORRESPONDING CLASS-INTERVAL. By WILLFORD I. KING AND LUCILE LAWRENCE 435

ECONOMIC AND SOCIAL SURVEY OF LONDON (440); FREIGHT TRAFFIC OF CALIFORNIA CITIES (440); A GRAPHIC DEVICE FOR ARITHMETIC OR GEOMETRIC INTERPOLATION (441); PROGRESS OF THE CENSUS (442); MISCELLANEOUS NOTES (445); MEMBERS ADDED (449).

REVIEWS:

Dublin: *Health and Wealth*, by A. Newsholme (452); Mills: *The Behavior of Prices*, by T. O. Yntema (453); Crum: *Advertising Fluctuations, Seasonal and Cyclical*, by F. E. Croxton (455); Dorau and Hinman: *Urban Land Economics*, by J. V. Van Sickle (456); Darmois: *Statistique Mathematique*, by J. R. Musselman (457); Wardwell: *An Investigation of Economic Data for Major Cycles*, by A. Achinstein (459); Newsholme: *Health Problems in Organized Society*, by E. Sydenstricker (462); Schmalz: *Standard Departmental Stock-Sales Ratios for Department Stores*, and Griffen: *Sales Quota Systems*, by H. Secrist (463); Harada: *Labor Conditions in Japan*, by W. B. Catlin (464); National Industrial Conference Board: *Employee Stock Purchase Plans in the United States*, by J. D. Brown (466); Carver and Lester: *This Economic World and How It May Be Improved*, and Mazur: *American Prosperity: Its Causes and Consequences*, by A. L. Bernheim (468); Burgess: *The Reserve Banks and the Money Market*, by G. W. Dowrie (470); Seymour: *The British Employment Exchange*, by S. M. Harrison (472); *International Yearbook of Agricultural Statistics*, by O. L. Dawson (474); Bogardus: *The New Social Research*, by W. F. Ogburn (474).

RECENT LITERATURE 476

Published Quarterly by the American Statistical Association

RUMFORD PRESS BUILDING, CONCORD, N. H.

Editorial Office, Columbia University, New York City

Entered at the post-office, Concord, N. H., as second-class mail matter

Acceptance for mailing at the special rate of postage provided for in Section No. 1103, Act of October 3, 1917, authorized April 29, 1922

Officers of the American Statistical Association

ORGANIZED NOVEMBER 27, 1839

President, Carl Snyder

Vice-Presidents, Seymour L. Andrew, Raymond Pearl, Ralph G. Hurlin

Counsellors, James W. Glover, W. Randolph Burgess, R. H. Coats

Secretary-Treasurer, Willford I. King,
Room 530, Commerce Bldg., New York University,
236 Wooster Street, New York, N. Y.

District Secretaries, Robert W. Bachelor,
1437 Bancroft Way, Berkeley, Calif.
F. L. Carmichael,
University of Denver, Denver, Colo.
John H. Cover,
University of Pittsburgh, Pittsburgh, Pa.
H. B. Flinkers,
Federal Reserve Bank of Cleveland, Ohio
Ira N. Frisbee,
331½ North Beverly Drive, Beverly Hills, Calif.
A. Heath Onthank,
George Harrison Phelps, Inc., Detroit, Mich.
Roswell F. Phelps,
Division of Statistics, Massachusetts Dept. of Labor and Industries, Boston, Mass.
S. W. Russell,
Commercial Research Department,
Swift and Company, Chicago, Ill.
C. D. Simmons,
University of Texas, Austin, Texas
Woodlief Thomas,
Federal Reserve Board, Washington, D. C.
Harry S. Will,
66 Fallis Road, Columbus, Ohio

Editor, Frank Alexander Ross,
Columbia University, New York City

Associate Editors, Edmund E. Day, Wesley C. Mitchell, William F. Ogburn, Walter F. Willcox
Leo Wolman,
30 Fifth Avenue, New York City

NEW SERIES, NO. 164 (VOL. XXIII) DECEMBER, 1928

JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

Formerly the Quarterly Publication of the American Statistical Association

THE LOGISTIC CURVE AND THE PREDICTION OF THE POPULATION OF THE CHICAGO REGION [1]

By Ardis T. Monk and Helen R. Jeter, *The University of Chicago*

In view of the recent discussion [2] of the theory of population growth expressed in terms of the logistic curve the results of a practical effort to apply the curve to the predicton of the future population of the Chicago region may be of some general interest.

The task of predicting the population was undertaken by the Local Community Research Committee of the University of Chicago at the request of the Chicago Commonwealth Club and the Chicago Regional Planning Association. The latter organization includes within its sphere of activities a territory of about 7,817 square miles in the immediate vicinity of Chicago. The area extends over parts of three states, Illinois, Indiana, and Wisconsin, and includes fifteen separate counties.[3]

Confronted with the practical problem of determining how large a population was likely, a few decades hence, to use the various facilities of the area, the committee undertook to predict by several different methods the probable growth of population in this region. Since the

[1] The authors are indebted to the Local Community Research Committee of the University of Chicago for permission to publish this article, and to Professor Henry Schultz for his great assistance and interest.

[2] No attempt is made here to set forth the theory of the logistic curve, but the reader is referred to the following expositions and discussions:

Raymond Pearl, *Studies in Human Biology*, pp. 523–639.

Raymond Pearl and Lowell J. Reed, *Predicted Growth of New York and Its Environs*. Published by Plan of New York and Its Environs, 1923.

Raymond Pearl, *The Biology of Population Growth*.

G. U. Yule, "The Growth of Population and the Factors Which Control It," *Journal of the Royal Statistical Society*, Vol. 88, January, 1925.

A. Lotka, *Elements of Physical Biology*, Chapter VII, pp. 64–76.

Sir George H. Knibbs, "Laws of Growth of a Population," this Journal, December, 1926, and March, 1927.

Harold Hotelling, "Differential Equations Subject to Error, and Population Estimates," this Journal September, 1927, pp. 283–315.

[3] These counties are Cook, DuPage, Grundy, Kane, Kankakee, Kendall, Lake, McHenry, and Will in Illinois; Lake, LaPorte, and Porter in Indiana; and Kenosha, Racine, and Walworth in Wisconsin.

logistic curve had been used with apparent success in the prediction of population for the New York region,[1] this was among the first methods applied. The results for the Chicago region were, however, of such a character that they were not included in the report recently published.[2] They are presented here as an interesting set of experiments in fitting a logistic to population data.

It may be questioned at the outset whether or not this region is a finite area such as is required in the hypotheses of the theory underlying the logistic curve.[3] Neither mountain barriers nor desert wastes shut it off from the next counties which happen not to be included in the "region." Nevertheless, it undoubtedly as nearly fulfills the requirement of a restricted area as does the New York region to which the method was successfully applied. It is, in short, a well-defined territory containing many diverse political districts but considerable economic unity and therefore may be as worthy of separate treatment as any other subdivision of the earth's surface. Few political subdivisions of the earth now are actually economically self-sufficient. Since the logistic curve has been applied to the population of many cities whose limits cannot be less artificial than those of the county boundaries surrounding the region of Chicago, we are perhaps justified in assuming that the region is a suitable one.

A further objection may be made upon the ground that the territory is not homogeneous in its character of growth and that no claim is made by anyone that the total population of an area composed of parts that behave in different ways will follow a general law of population growth. Several of the fifteen counties present considerable evidence of rapid industrial growth. On the other hand, a large percentage of the area of the region is agricultural and in the agricultural portions many of the districts show declining populations. An effort was made to eliminate this factor of declining population by experimenting with population figures for Cook County only, which contains a large proportion of the population of the region and relatively small areas of decreasing population. The results, however, seem to be not far different from those for the entire region. Most of the calculations, therefore, have been made for a larger area.

Another limitation to the application of any general law to the population of this region is fairly obvious; the number of actual counts of population is very small. Only seven censuses, beginning with 1860, furnish actual figures for the entire fifteen counties. For twelve

[1] Raymond Pearl and Lowell J. Reed, *Predicted Growth of Population for New York and Its Environs*. Published by Plan of New York and Its Environs.

[2] Helen R. Jeter, *Trends of Population in the Region of Chicago*, The University of Chicago Press, 1927.

[3] Raymond Pearl, *Studies in Human Biology*, p. 567.

counties,[1] however, census figures are available for nine decades beginning with 1840. Since these twelve counties are contiguous and contain 98.0 per cent of the population of the entire region, most of the calculations have been made for them rather than for the entire region. While even nine decades constitute a short period upon which to base predictions for the next century or two, they nevertheless represent practically the entire history of civilized habitation in this region.[2] Applications have been made, although admittedly without much confidence, to even shorter periods of observation.[3]

The first attempt to fit a logistic curve to the census data for the population of the entire region from 1860 to 1920 and to predict the future population by extrapolation was an application of the simple symmetrical three constant curve, suggested by Professor Pearl,[4] and represented by the equation,[5]

$$y = \frac{k}{1+e^{a_0+a_1 x}}. \tag{1}$$

This curve is fitted to the actual data by passing it through any three points equidistant in time.[6] Since no general principles have been laid down for the selection of these points it is a matter of choice on the part of the statistician. One of the most logical selections would seem to be points that include the whole period of observation, *i. e.*, the first, last, and middle census years. The curve fitted to the data for these years, 1860, 1890, and 1920 has the following equation:[7]

$$y = \frac{6.040}{1+e^{2.111-.05359x}}.$$

From this equation we are brought to the somewhat startling conclusion that the asymptote or maximum population of this region is 6,040,000, and that this figure will not be reached until about the year 2050. The population of the region given by the census of 1920 was 3,858,818. Since 1890 the population has been increasing at an aver-

[1] These twelve counties are Cook, DuPage, Kane, Lake, McHenry, and Will in Illinois; Lake, LaPorte and Porter in Indiana; and Kenosha, Racine and Walworth in Wisconsin. The fifteen counties include in addition to these, Grundy, Kankakee and Kendall counties in Illinois.

[2] The population of the entire region in 1830 was probably less than 1,000.

[3] See predictions for Belgium, Hungary and Japan in Raymond Pearl, *Studies in Human Biology*, pp. 596, 609 and 625.

[4] *Studies in Human Biology*, p. 578.

[5] The unit of measure of y and k (population) is one million, and the unit of time one year throughout this paper.

[6] Because of the fact that the actual census intervals are unequal the data have been adjusted by logarithmic interpolation to give population figures for January 1 of each year. For these figures see Appendix IV.

[7] This is curve A of Table I.

age rate of about 750,000 per decade. This equation would lead us to expect, however, that the total increase during the one hundred and thirty years following 1920 would be less than the increase during the preceding period of only thirty years. This rather surprising result led to experiments with applications of the same equation to data for different sets of years. The results of a few of the possible combinations for the fifteen counties are shown in Table I, lines A to F inclusive and in Chart I. The experiments in fitting equation (1) to the data for twelve counties are also summarized in Table I, lines G to L inclusive, and two of the corresponding curves are shown in Chart II.

TABLE I

EFFECT OF CHOICE OF POINTS ON CURVE IN THE FORM $y = \frac{k}{1+e^{a_0+a_1x}}$

	Group of points through which curve is passed	Constants			Year of point of inflection	$\frac{\Sigma(\Delta y)^2}{n}$ *
		k in millions	a_0	a_1		
	Fifteen counties (population in 1920, 3,858,818)					
A	1860–1890–1920	5.991	2.654	−0.05411	1909	0.00172
B	1870–1895–1920	6.040	2.111	−0.05360	1909	0.00168
C	1875–1895–1915	5.262	1.742	−0.05894	1904	0.00299
D	1880–1900–1920	5.186	1.501	−0.06420	1903	0.00358
E	1870–1890–1910	5.674	2.044	−0.05440	1908	0.00345
F	1890–1905–1920	6.397	1.118	−0.05123	1912	0.00358
	Twelve counties (population in 1920, 3,785,224)					
G	1840–1880–1920	4.495	4.470	−0.07680	1898	0.01454
H	1855–1885–1915	5.559	3.072	−0.05803	1908	0.00297
J	1860–1890–1920	5.690	2.716	−0.05671	1908	0.00198
K	1870–1895–1920	5.802	2.160	−0.05580	1909	0.00206
L	1850–1885–1920	5.471	3.431	−0.06058	1907	0.00215
M	9 points, 1840–1920, inclusive	3.783	4.133	−0.07792	1893	0.02720
N	9 points, 50-year average increases	3.980	4.177	−0.07387	1896	0.02129
O	8 points, 30-year average increases	5.470	3.536	−0.05863	1910	0.03753

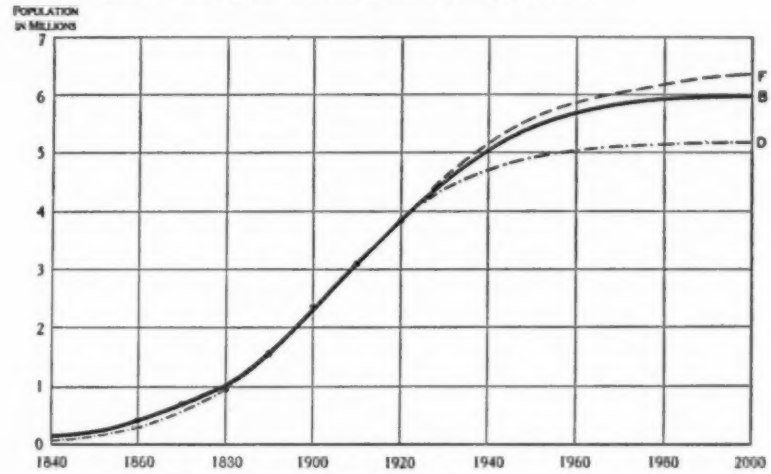
* $\frac{\Sigma(\Delta y)^2}{n}$ is the mean of the squares of the differences between the census counts of population in millions, and the populations computed from the equation.

There are evidently many possible answers to our question as to what the population will be at some future date. The asymptote or upper limit to the population of the twelve counties ranges, in the few experiments shown, from about four and one-half million to nearly six million, the date of the point of inflection ranges from 1893 to 1910, and the date at which the maximum will be reached ranges from about 1940 to about 2000. Which of these or of the results of other possible experiments is the best answer?

CHART I

THE LOGISTIC CURVE IN THE FORM $y = \frac{k}{1+e^{a_0+a_1 x}}$ FITTED TO POPULATION DATA[1] FOR FIFTEEN COUNTIES IN THE CHICAGO REGION FOR THREE DIFFERENT SETS OF YEARS[2]

The constants of these curves are shown in Table I, lines B, D and F.



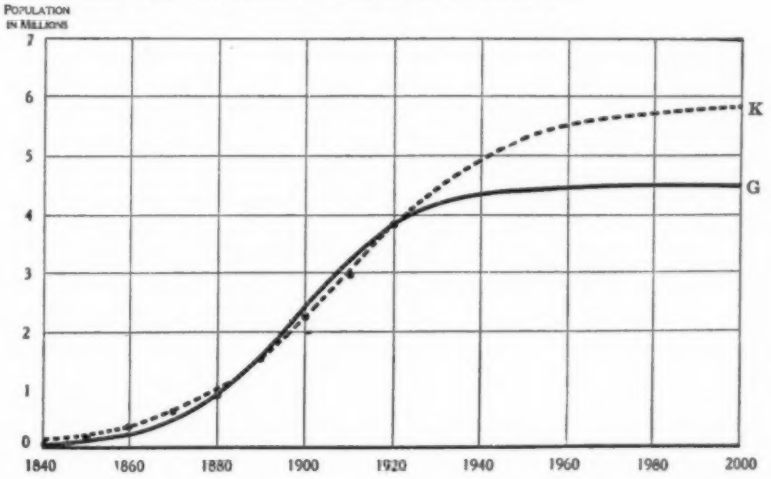
[1] Observed populations are represented by dots on chart.

[2] The part of curve F covering the period from 1840 to 1925 coincides (on the small scale of this graph) with curve B.

CHART II

THE LOGISTIC CURVE IN THE FORM $y = \frac{k}{1+e^{a_0+a_1 x}}$ FITTED TO POPULATION DATA FOR TWELVE COUNTIES IN THE CHICAGO REGION FOR TWO DIFFERENT SETS OF YEARS

The constants of these curves are shown in Table I, lines K and G.



Two criteria have been kept in mind in judging how satisfactorily any given curve represents the data. One of these is the "fit" of the curve to the whole range of observed points, as measured by $\frac{\Sigma(\Delta y)^2}{n}$, the mean of the squares of the differences between the populations computed from the formulae and the observed populations at the census dates. According to this criterion the curve of "best fit" is one for which $\frac{\Sigma(\Delta y)^2}{n}$ is smallest.

The other criterion is the purely aesthetic judgment, by inspection of the graph, as to how the computed curve follows the probable future trend of the observations. In general, the two criteria will amount to much the same thing, but it is conceivable that the observed points may be fairly well fitted by a curve which, extrapolated beyond those points, exhibits a trend quite different from the apparently obvious trend of the observations.

The curves of "best fit" in Table I, according to the first criterion, are B for the fifteen counties (shown in Chart I) and J for the twelve counties. On the small scale of Chart II, curve J would differ very slightly from curve K shown there. It should be noted that neither of these curves was obtained from the first, middle and last years of observation and that our original assumption that passing the curve through the data for these years would give the best results is not borne out. Other than this we can draw no conclusion with regard to selection of points except that for these particular data the inclusion of the year 1880 seems for some reason to produce a low asymptote and a relatively worse fit.

Such a degree of variability in the results of fitting the curve in this form by passing it through only three points suggested that better results might be obtained by using more data.[1] This may be done according to Professor Pearl's suggestion [2] of a five constant curve, passed through five points, or by two methods, suggested by Professor Yule,[3] of fitting the three constant curve to all the data.

One of these methods suggested by Professor Yule makes use of all the available data (provided the number of censuses available is a multiple of 3) by summing the reciprocals of the population data in

[1] The usual least squares method of fitting a curve to a set of points is to find the unknown constants of the curve by the solution of a set of "normal equations" which are derived from all the observed data; but the method requires the equation to be linear with respect to these constants. The constants do not enter linearly into the equation of a logistic, and hence this direct method of using all the data is not available to us. *Corrections* to the constants may be made, however, by the least squares method. See Pearl, *loc. cit.*, p. 578, and *infra.*, p. 15.

[2] *Studies in Human Biology*, p. 573.

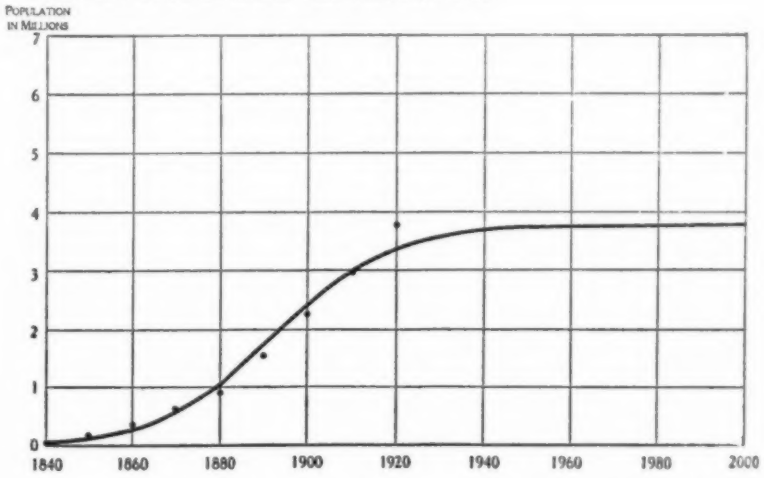
[3] *Journal of the Royal Statistical Society*, Vol. 88, pp. 51 and 53.

three successive groups. The three constant form of the curve is then fitted by using these three sums instead of three single population figures equidistant in time.[1] This method was applied to the data for the twelve counties from 1840 to 1920, summing the populations in groups of three each. The results are shown in Table I, line M, and in Chart III. This method obviously gives undue weight to the earlier populations,[2] and this fact probably accounts for the impossibly low asymptotic population in this case, only 3,783,000, which is less than the actual count of population in 1920.

CHART III

THE LOGISTIC CURVE IN THE FORM $y = \frac{k}{1+e^{a_0+a_1x}}$ FOR THE CHICAGO REGION (TWELVE COUNTIES) FITTED BY YULE'S "METHOD OF SUMS OF RECIPROCALS," TO CENSUS DATA FOR THE YEARS 1840 TO 1920, INCLUSIVE

The constants of this curve are shown in Table I, line M.



The other method of fitting suggested by Professor Yule, the "Method of Percentage Increases," is based upon the fundamental assumption from which the equation for the logistic was derived by Verhulst. In Yule's words the reasoning of Verhulst is stated as follows:[3]

> If a population is expanding freely over unoccupied country, the percentage rate of increase is constant. If it is growing in a limited area, the percentage rate of increase must tend to get less and less as the population grows, so that the

[1] See Appendix I B for details of this method.

[2] Professor Henry Schultz suggests that a proper method of weighting the sums of reciprocals would remove this objection.

[3] *Journal of the Royal Statistical Society*, Vol. 88, p. 4.

percentage rate of increase is some function of the population itself, which falls continuously as the numbers of the population rise. But what function? In the absence of any precise knowledge we had better take the simplest form of the function that fulfills the required conditions, say $m-np$, where p is the population and m and n are constants. Writing down the differential equation which is implied by this argument, the solution is the required probable form for the law of growth of a population confined to a given area.

In other words, the logistic curve depends upon a linear relationship between the percentage rate of increase of the population and the population itself. Yule suggests that the constants of the logistic, which he writes in the form

$$y = \frac{L}{1+e^{\frac{\beta-t}{\alpha}}}$$

may be determined by the straight line fitted to the points which represent percentage increases over finite intervals of time plotted against the population at the ends of these intervals. The X intercept of this straight line gives the asymptotic population,[1] (L in Yule's notation, k in Pearl's). The results of this method of fitting to the data for the Chicago region are shown in Table I, lines N and O, and in Charts IV, V, and VI.

Chart IV shows the points whose coördinates are the average percentage increases in population over fifty year intervals (the Y's), and the populations at the ends of these intervals (the X's), and two straight lines fitted to the points by the method of least squares.[2] Chart V is similar except that the percentage increases in population were averaged over thirty year intervals.

Although the points in Chart IV deviate from a straight line in much the same way as in Chart V, no significance would be attached to this deviation, which might be purely random, were it not for the fact that when similar sets of points are plotted for ten, twenty, and forty year averages, there is the same systematic deviation from a straight line and the trend of the points in every case seems not linear, but distinctly curved.

Curves N and O (Table I, lines N and O and Chart VI), which are the logistic curves derived from the straight lines shown in Charts IV (B) and V, show that this method of fitting to the data for the Chicago region gives particularly poor results. The fit is distinctly worse than

[1] For the details of the determination of α and β, see Appendix I C.

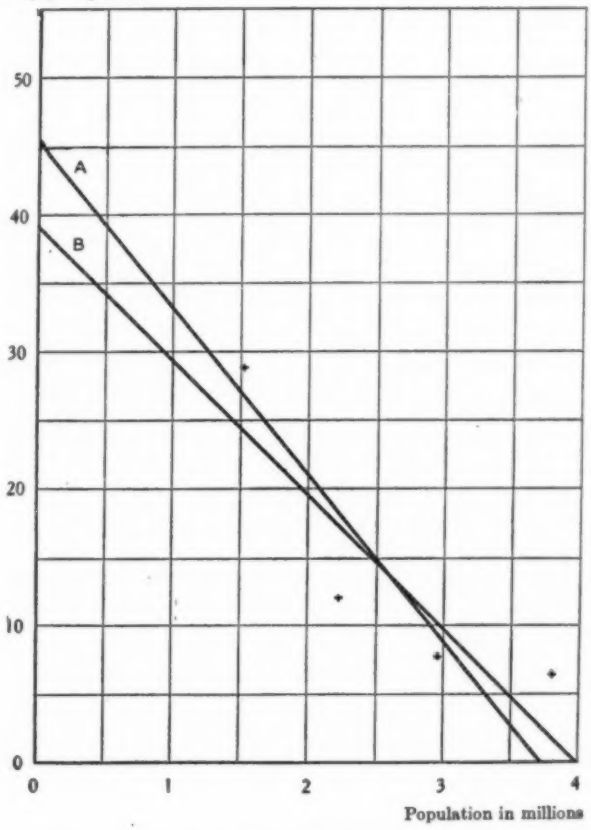
[2] Line B is so determined as to minimize the sum of the squares of the vertical deviations of the points from the line, and line A, to minimize the sum of the squares of the perpendicular deviations of the points from the line. (See Appendix I C.) It happens that the line minimizing the sum of the squares of the horizontal deviations coincides almost exactly with line A. In Chart V these two lines are indistinguishable.

CHART IV

PERCENTAGE INCREASES IN POPULATION IN THE CHICAGO REGION (TWELVE COUNTIES) DURING FIFTY YEAR INTERVALS PLOTTED AGAINST THE POPULATION AT THE ENDS OF THE FIFTY YEAR INTERVALS, AND TWO STRAIGHT LINES FITTED TO THESE POINTS

Line B is so determined as to minimize the sum of the squares of the vertical deviations of the points from the line.

Line A is so determined as to minimize the sum of the squares of the perpendicular deviations of the points from the line.



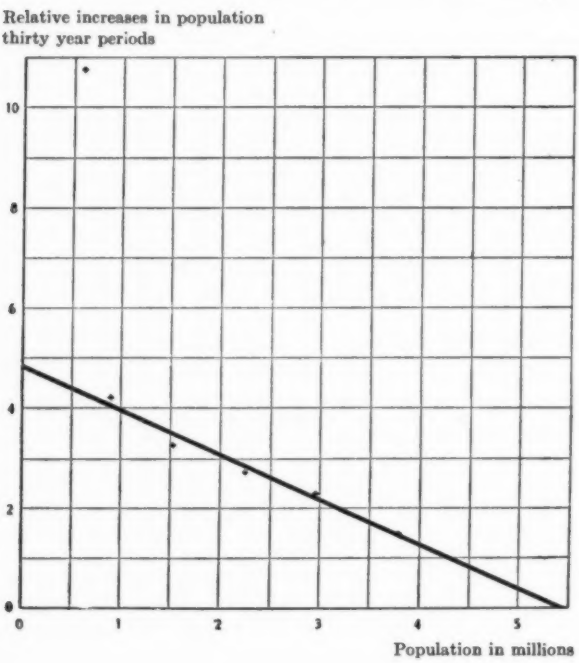
* To express the ordinates in percentages, multiply by one hundred.

that of curves obtained by the simple three point fitting, and the trend of the curve, inside the range of observation as well as beyond, shows the same kind of departure from the obvious trend of the points as does

CHART V

PERCENTAGE INCREASES IN POPULATION IN THE CHICAGO REGION (TWELVE COUNTIES) DURING THIRTY YEAR INTERVALS PLOTTED AGAINST THE POPULATION AT THE ENDS OF THE THIRTY YEAR INTERVALS, AND A STRAIGHT LINE FITTED TO THESE POINTS BY THE METHOD OF LEAST SQUARES

(Note: In fitting this line the sum of the perpendicular deviations has been made a minimum and the first point has been disregarded in fitting.)



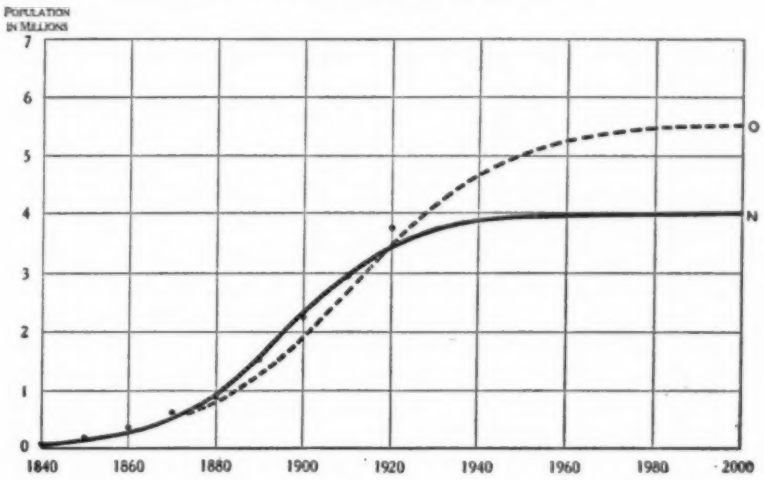
curve M, Chart V, discussed earlier. Curve O, which follows from the straight line fitted, as an experiment, by ignoring the point which seemed to spoil the linearity of relationship between the percentage increases of population and the population itself, can hardly be said to fit at all.

If the trend of the points which represent the percentage increases in population is actually non-linear, a logistic could hardly be expected to represent very accurately the growth of this particular population, since, as we have seen above, a logistic is derived on the assumption that the percentage rate of increase of a population is a linear function of that population.

CHART VI

THE LOGISTIC CURVE IN THE THREE CONSTANT FORM $y=\frac{k}{1+e^{a_0+a_1x}}$ FOR THE CHICAGO REGION (TWELVE COUNTIES) FITTED BY YULE'S "METHOD OF PERCENTAGE INCREASES" TO DATA FOR THE YEARS 1840 TO 1920, INCLUSIVE

Curve N depends upon percentage increases averaged over fifty year periods; Curve O depends upon percentage increases averaged over thirty year intervals.



Another method of utilizing more of the data, the five constant curve

$$y=\frac{k}{1+e^{a_0+a_1x+a_2x^2+a_3x^3}} \tag{2}$$

suggested by Professor Pearl, should be more reliable than the three constant form since it makes use of more points, and is also free from the restriction of symmetry. A curve with five constants, however, is so flexible that when it is fitted to five points spread over only seven or nine decades there can be no certainty about its superiority.

In two attempts to fit this form of the curve to the data for the fifteen counties the five points were taken at ten year intervals from 1880 to 1920 inclusive, and at fifteen year intervals from 1860 to 1920 inclusive. The k, surprisingly enough, was found to be not only less than the 1920 population, but less than the population for 1910 in the first trial, and for 1905 in the second. The form of the equation is such that these small figures for k rendered the curves imaginary for the data for the fifteen counties.

Fitted to the data for the twelve counties, however, the five

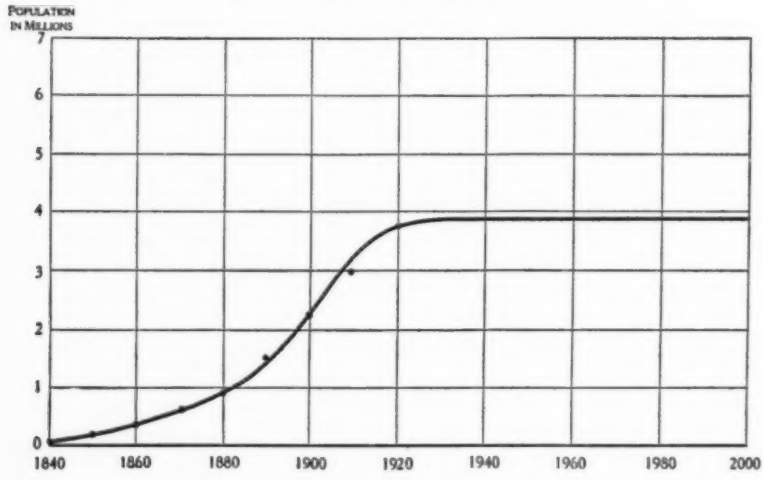
constant curve appears as presented in Chart VII, with constants as follows:

$$k = +3.887 \text{ millions}$$
$$a_0 = +4.323$$
$$a_1 = -0.1469$$
$$a_2 = +0.002862$$
$$a_3 = -0.00002834$$

The predictions from this curve seem improbable since they would indicate that the population of the region could never increase more than 100,000 above the figure for 1920 and would remain stationary after 1925 at about 3,900,000.

CHART VII

THE LOGISTIC CURVE IN THE FIVE CONSTANT FORM $y = \frac{k}{1+e^{a_0+a_1x+a_2x^2+a_3x^3}}$
FITTED TO THE POPULATION OF THE CHICAGO REGION (TWELVE COUNTIES)



A further attempt was made to fit the data with still another variation, a two cycle curve whose upper cycle is the four constant form,[1]

$$y = d + \frac{k}{1+e^{a_0+a_1x}} \qquad (3)$$

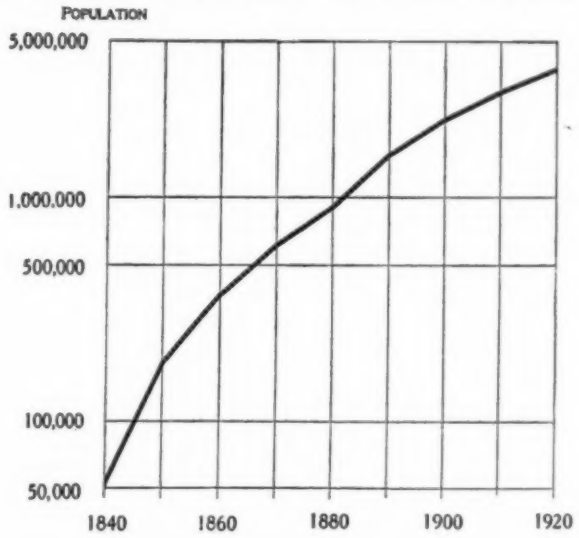
following a first cycle represented by the three constant form. This two cycle form of the curve seemed worth trying since inspection of the observed population curve indicates a possible change of cycle about

[1] *Studies in Human Biology*, p. 575.

1880. The data plotted on a logarithmic scale as in Chart VIII show this possibility of a break, although it becomes somewhat smoothed out on the small scale of this graph. The fact that the inclusion of the population figure for 1880 in preceding attempts to fit the three constant form of the curve 'ent added weight to the idea that the four constant or two cycle form of the curve might represent better the growth of population in this region.

CHART VIII

POPULATION OF THE CHICAGO REGION (TWELVE COUNTIES) 1840–1920 PLOTTED ON SEMI-LOGARITHMIC SCALE



The initial difficulty in using the four constant curve arises from the fact that the fourth constant, d, according to Professor Pearl's custom, is not determined by direct solution, but is obtained by a graphical approximation.[1] There is thus a large element of choice in its determination.[2] Since the lower asymptote of the second cycle may fall below the upper asymptote of the first cycle, the value of d may fall anywhere between zero and the upper asymptote of the first cycle. The curve of best fit for the cycle apparently ending about 1880 was found to have an asymptote of 1,100,000. The value of d for the second cycle may therefore fall between 0 and 1,100,000. Since the four constant curve is symmetrical, the lower the value of d the higher

[1] The authors are grateful to Professor Pearl for his replies to inquiries concerning this point.

[2] For the determination of d by direct algebraic solution see Appendix III to this paper.

the asymptote becomes. For these data, moreover, the lower the value of d the better the fit of the curve from 1890 to 1920, and when $d=0$ the best fit is obtained. The two cycle curve, therefore, as shown in Chart IX, is the result of combining a three constant curve fitted to the data for 1860, 1870 and 1880 and a four constant curve (with $d=0$, which therefore reverts to three constants) fitted to the data for 1890, 1905 and 1920. These two curves intersect twice between the years 1855 and 1875. In the graph, therefore, the curve from 1840 to 1860 is the curve of the lower cycle, the curve from 1880 to 2000 is the curve of the upper cycle, and between 1860 and 1880 the curve is the arithmetic mean of the curves of the two cycles. The two curves are so close together in the interval from 1860 to 1880 that it seemed not worth while for our purposes to use any more extended blending process. The two cycle method applied to these data, therefore, gives a curve which differed too little from the simple one cycle curve to make the extra labor worth while in this case.

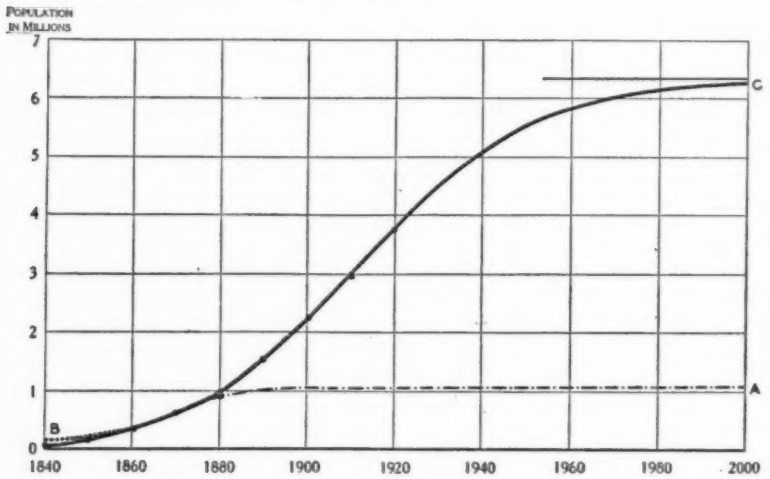
CHART IX

THE LOGISTIC CURVE IN THE TWO CYCLE FORM, FITTED TO THE POPULATION OF THE CHICAGO REGION (TWELVE COUNTIES)

A represents the curve of best fit in the form $y=\frac{k}{1+e^{a_0+a_1x}}$ for the lower cycle, or the period ending about 1880.

B represents the curve of best fit in the form $y=d+\frac{k}{1+e^{a_0+a_1x}}$ for the upper cycle, or the period from 1890 to 1920.

C represents the blended curve for the two cycles.



Since the four and five constant forms gave no better results, the next step was an effort to improve the fit of the three constant curves

by least squares corrections. By treating the values of one such set of constants as first approximations, a second approximation may be made by applying corrections to these values by the method of least squares, and presumably, a better fit will be obtained since the least squares corrections involve the use of all the data. These corrections, obtained by the method suggested by Professor Pearl,[1] and also by the general method described in any text on least squares,[2] were applied to the constants of the curves of best fit for both the fifteen and the twelve counties (curves B and J of Table I). The Pearson "method of false position" [3] was also applied to curve J for the twelve counties. These results are shown in Table II.

TABLE II

CONSTANTS OF THE EQUATION $y = \frac{k}{1+e^{a_0+a_1x}}$ CORRECTED BY THE METHOD OF LEAST SQUARES

	k in millions	a_0	a_1	$\frac{\Sigma(\Delta y)^2}{n}$
Fifteen counties				
From Table I, B	6.040	2.111	−0.05360	0.00168
B_1, Corrected by Pearl's method	6.362	2.196	−0.05300	0.00201
B_2, Corrected by the general method	5.999	2.038	−0.05602	0.00179
Twelve counties				
From Table I, J	5.690	2.716	−0.05670	0.00198
J_1, Corrected by Pearl's method	4.169	2.599	−0.06772	.....
J_2, Corrected by the general method	2.594	2.221	−0.06838	.....
J_3, Corrected by Pearson's "method of false position"	5.474	3.941	−0.05908	0.00152

It appears from Table II that, contrary to expectation, the constants corrected by the Pearl method and by the general method for the fifteen counties differ only slightly from those originally obtained and the fit of the curve is not so good. For the twelve counties the corrected constants differ considerably from the original.[4] The new asymptote is in fact so very low that the fit of the curve is obviously much worse than that of the original curve. Pearson's "method of false position," however, slightly improves the fit.

[1] *Studies in Human Biology*, p. 578.

[2] For example, O. M. Leland, *Practical Least Squares*, McGraw-Hill, p. 137.

[3] Karl Pearson, "On a General Theory of the Method of False Position," *Philosophical Magazine*, Series 6, Vol. 5, 1903, p. 658.

[4] The least squares method of arriving at a second approximation of the value of the constants assumes that second and higher order terms of a Taylor's expansion are negligible. The fact that the second approximations above are worse than the first, indicates that these higher terms are not negligible.

For the sake of completeness it seemed desirable to determine the probable errors of the constants of the logistic curves, but in view of the meager data for the Chicago region the validity of any probable error determinations for these constants would seem doubtful. The variation in the constants due to different "samplings" of the observations, as shown in Table I, is in itself, however, an indication of their probable errors.

The data here examined warrant no conclusions with regard to the general validity of the logistic curve as a law of population growth. It is possible, however, that although considerable confidence may be placed in the curve where it is applied to large areas with relatively large populations, that it is uncertain when applied to cities or small regions. This study was originally undertaken for the practical purpose of predicting the population for a relatively short period in the future, that is, from 1920 to 1950. For such prediction the machinery of the logistic curve was probably unnecessary and only the interest which attached to the variability of the results led to further and further experiments. If any conclusion may be ventured it is that for this particular region little confidence can be placed in extrapolations from the logistic. The validity of the simple three constant form seems more probable than that of the four and five constant forms. Extrapolations from even this simple curve, however, vary so widely among themselves and also from what may be considered on common sense grounds to be possible that the effort to arrive at practicable results by this mathematical method was finally abandoned by the committee in favor of the more empirical methods of reputable public service companies in the region. It may be of interest to compare in Table III the predictions for the year 1950 resulting from all these various methods.

Even in 1950, less than thirty years hence, all of the predictions from the logistic curve fall considerably below the predictions of the public service companies and the United States census method of straight line extrapolation. While this behavior of the curve should not in itself discredit the method in general, nor even in this particular case, it is apparent that certain characteristics of the population of this region prevent the application of a logistic. It is perhaps beyond the scope of this paper to attempt a discussion of why this is the case, but two factors in the situation have already been mentioned: First, that nine census counts are probably too few to depend upon, and second, that the percentage rate of increase is probably not a linear function of the population.

An additional factor may be mentioned in the case of the three

TABLE III

POPULATION OF TWELVE COUNTIES IN THE REGION OF CHICAGO PREDICTED FOR 1950 ACCORDING TO SPECIFIED METHODS

Source or method of prediction	Predicted population in 1950
Public service companies:	
Bell Telephone Company	6,857,400
National Electric Light Association	6,564,122
Several public service companies, combined	6,823,200 *
Mathematical formulae:	
Straight line through data for last three decades	6,023,000
Logarithmic curve $y = a + bx + cx^2 + d \log x$	7,900,000 †
Logistic curves:	
Three constant form $y = \frac{k}{1 + e^{a_0 + a_1 x}}$:	
Pearl's method of fitting, Table I, line G	4,412,000
Table I, line J	5,212,000
Table I, line K	5,275,000
Yule's method of fitting, Table I, line M	3,739,000 ‡
Table I, line N	3,905,000
Two cycle form	5,545,000
Five constant form $y = \frac{k}{1 + e^{a_0 + a_1 x + a_2 x^2 + a_3 x^3}}$	3,886,000

* This is the figure published in *Trends of Population in the Region of Chicago.*
† Approximately.
‡ This is less than the census figure for 1920.

constant curve. It is obvious from all the charts shown that the logistic curves for this region reach their points of inflection and consequently their asymptotes at comparatively early dates. This possibly may be explained by the fact that, in the case of the symmetrical three constant curves, a sharp rise from the lower asymptote necessarily involves an equally abrupt flattening of the curve toward the upper asymptote. Population in the Chicago region, beginning with almost zero in 1820, and reaching 3,785,000 in 1920, has grown so rapidly during this short period of time that the asymptotic value of a symmetrical logistic based on this period must necessarily be low.

To sum up, the history of growth of population in this region has not exhibited the necessary characteristics of a logistic: First, the period of observation is too short to draw sound conclusions; second, the rate of growth has been so rapid that a symmetrical curve would exhibit an equally rapid falling off in the rate of increase; and third, the percentage rate of growth may not be a linear function of the population itself. Hence a first order logistic could not be expected either to fit the observed data or to warrant prediction beyond the observed points by extrapolation.

APPENDIX I

FITTING THE THREE CONSTANT CURVE,

$$y=\frac{k}{1+e^{a_0+a_1x}}. \tag{1}$$

A. *Fitting to Three Points*

Pearl's solution [1] for the three constants k, a_0, and a_1, is made by passing the curve through three points (x_0, y_0), (x_1, y_1), and (x_2, y_2). On taking these points equidistant in time, and putting $x_0=0$, we get their coördinates (since $x_2-x_1=x_1-x_0=x_1$ whence $x_2=2x_1$), $(0, y_0)$, (x_1, y_1), $(2x_1, y_2)$, and the solutions for the constants, from (1)

$$k=\frac{2y_0y_1y_2-y_1^2(y_0+y_2)}{y_0y_2-y_1^2} \tag{4}$$

$$a_0=\log_e\frac{k-y_0}{y_0} \tag{5}$$

$$a_1=\frac{1}{x_1}\log_e\frac{y_0(k-y_1)}{y_1(k-y_0)}. \tag{6}$$

In this paper the unit of time used was one year.

The value of a_0, it will be observed, depends on the year taken as origin. If the solution for the constants is made without setting $x_0=0$, but with $x_2-x_1=x_1-x_0=t$, the coördinates of the three points are (x_0,y_0), (x_0+t, y_1) and (x_0+2t, y_2). The solutions for k and a_1 are unchanged, but that for a_0 becomes

$$\begin{aligned} a'_0&=\log_e\frac{k-y_0}{y_0}-a_1x_0 \\ &=a_0-a_1x_0 \end{aligned} \tag{7}$$

where the constant is a_0 when referred to the origin zero, and a'_0 when referred to the origin x_0.

When a second approximation to the values of these constants is made by Pearson's "Method of False Position," [2] using the constants from lines H, J, K, and L of Table I as "trial solutions," in the sense of Pearson's paper, it is necessary that the a_0's of these solutions be referred to a common origin by the transformation given in (7).

[1] *Studies in Human Biology*, p. 578.

[2] Karl Pearson, "On a General Theory of the Method of False Position," *Philosophical Magazine*, 6 Series V, 1903, p. 658.

B. *Yule's Method of Fitting by "Sums of Reciprocals"* [1]

This method of fitting makes use of any multiple of three observations. Yule writes equation (1) in the form

$$y=\frac{L}{1+e^{\frac{\beta-t}{\alpha}}} \tag{8}$$

where the unit of t is the census interval used, usually ten years. A comparison of equations (1) and (8) shows that Pearl's constants are related to Yule's as follows:

$$\left.\begin{aligned} k&=L \\ a_1&=-\frac{1}{\alpha} \\ a_0&=\frac{\beta}{\alpha} \end{aligned}\right\} \tag{9}$$

when the same time unit is used. In these computations the year was the unit used except when fitting by Yule's methods, when the unit was ten years. Consequently, when translating the constants found by Yule's methods, into the corresponding Pearl constants, we must write the second of equations (9), $a_1=-\frac{1}{10\alpha}$.

The year of the point of inflection of the curve, given in Tables I and II, is found by adding β, in years, to x_0, β being the interval between x_0 and the point of inflection.

Yule finds the constants L, α, and β as follows: Call the first observation date zero time, and the census interval the unit of time. Sum the reciprocals of the populations (at equidistant time intervals) in three successive groups S_1, S_2, and S_3, of r each; whence

$$\left.\begin{aligned} S_1&=\frac{r}{L}+\frac{C}{L}e^{\frac{\beta}{\alpha}} \\ S_2&=\frac{r}{L}+\frac{C}{L}e^{\frac{\beta-r}{\alpha}} \\ S_3&=\frac{r}{L}+\frac{C}{L}e^{\frac{\beta-2r}{\alpha}} \end{aligned}\right\} \tag{10}$$

where

$$C=\frac{1-e^{\frac{-r}{\alpha}}}{1-e^{\frac{-1}{\alpha}}}.$$

[1] G. U. Yule, "The Growth of Population," *Journal of the Royal Statistical Society*, Vol. 88, January, 1925, p. 51.

Put
$$\left.\begin{aligned} D_1 = S_1 - S_2 = \frac{C}{L}\left(1 - e^{\frac{-r}{\alpha}}\right)e^{\frac{\beta}{\alpha}} \\ D_2 = S_2 - S_3 = \frac{C}{L}\left(1 - e^{\frac{-r}{\alpha}}\right)e^{\frac{\beta - r}{\alpha}} \end{aligned}\right\} \tag{11}$$

whence
$$e^{\frac{r}{\alpha}} = \frac{D_1}{D_2}. \tag{12}$$

Also
$$\frac{D_1^2}{D_1 - D_2} = \frac{C}{L}e^{\frac{\beta}{\alpha}} \tag{13}$$

and therefore
$$\frac{r}{L} = S_1 - \frac{D_1^2}{D_1 - D_2}. \tag{14}$$

Equations (12), (13), and (14) give α, β, and L, respectively.

C. *Yule's "Method of Percentage Increases"* [1]

By this method of fitting, the fractional increases in population [2] over h census intervals are plotted as ordinates against the populations at the ends of the intervals [3] as abscissae, and to these points is fitted, by the method of least squares, a straight line,

$$Y = AX + B \tag{15}$$

where
$$A = \frac{e^{\frac{h}{\alpha}} - 1}{L} \tag{16}$$

and
$$B = e^{\frac{h}{\alpha}} - 1$$

The X-intercept of the line (15) gives L, the limiting value of the population, *i. e.*, the population (X) when the fractional increase of population (Y) has become zero. From the Y-intercept, B of equation (15), is determined, using the second of equations (16).

β is determined from the first of equations (10) when for S_1, is substituted the sum of the reciprocals of the populations for the whole series of observations, and, for r, the entire number of observations used.

The straight line of equation (15) may be fitted, as pointed out earlier, by the method of least squares so as to minimize the sum of the squares of the distances from the points to the line measured parallel

[1] Yule, *loc. cit.*, p. 52.

[2] $\frac{y_{t+h} - y_t}{y_t} = Y$.

[3] $y_{t+h} = X$.

either to the Y-axis or to the X-axis. (The former is the usual interpretation.) In the terminology of correlation theory the line fitted so as to minimize the Y-deviations is the regression of Y on X; the second, minimizing the X-deviations, the regression of X on Y. Their equations are, respectively,

$$Y=\overline{Y}+r\frac{\sigma_y}{\sigma_x}(X-\overline{X}) \tag{17}$$

$$Y=\overline{Y}-\frac{\sigma_y}{\sigma_x r}(X-\overline{X}) \tag{18}$$

where
$$\begin{cases} \overline{X}=\dfrac{\Sigma X}{n} \\ \overline{Y}=\dfrac{\Sigma Y}{n}, \end{cases}$$

r = correlation coefficient between X and Y, and the σ's, the standard deviations.

A third method,[1] fits the line so as to minimize the sum of the squares of the perpendicular distances from the points to the line. The equation of the line fitted in this manner is

$$Y=\overline{Y}+(X-\overline{X})\tan\theta \tag{19}$$

where
$$\tan 2\theta=\frac{2r\sigma_x\sigma_y}{\sigma_x^2-\sigma_y^2}. \tag{20}$$

These three methods of fitting the straight line involve the assumptions that, in the first line, the deviations of the points from the line are due entirely to errors in the Y's, (*i. e.*, in the fractional increases in population); in the second, that the deviations are due entirely to errors in the X's (*i. e.*, in the actual or estimated populations for January 1); and, in the third, that the deviations are due to errors both in the X's and the Y's.

APPENDIX II

FITTING THE FIVE CONSTANT CURVE

The determination of the constants[2] of the Pearl five constant equation,

$$y=\frac{k}{1+e^{a_0+a_1x+a_2x^2+a_3x^3}} \tag{2}$$

[1] E. C. Rhodes, "On Lines and Planes of Closest Fit," *Philosophical Magazine*, 7 Series, III (February, 1927), p. 357. For a very full bibliography on this subject see Henry Schultz, *Statistical Laws of Demand and Supply with Special Application to Sugar*, pp. 66–68.

[2] Pearl, *loc. cit.*, pp. 576–7.

fitted to the five points, $(0, y_0)$, (x_1, y_1), $(2x_1, y_2)$, $(3x_1, y_3)$ and $(4x_1, y_4)$ is as follows:

The k, in terms of the observed y's, is given by the equation

$$y_1^4 y_3^4 (k-y_0)(k-y_2)^6(k-y_4) = y_0 y_2^6 y_4 (k-y_1)^4 (k-y_3)^4, \qquad (26)$$

which reduces to seventh degree on expanding and combining terms. In the fitting to several different sets of five observations for the Chicago region, there has never been any uncertainty as to which root of this equation for k was a valid one for the problem at hand.

The a's are determined in terms of β's defined by the equation

$$\beta_i = \log\left(\frac{k-y_i}{k-y_0} \cdot \frac{y_0}{y_i}\right), \qquad (27)$$

where $i = 1, 2, 3$, each a being a function [1] of β_1, β_2, and β_3.

An attempt was made to fit this five constant equation to the data for the fifteen counties, taking the five points at ten year intervals from 1880 to 1920. In this case, all the roots of k in equation (26) are less than 3 millions; but y_3, the 1910 population, is 3.02 millions. Hence in equation (27) $k-y_3$ is negative, and consequently β_3 is imaginary. Since each of the a's is a function of β_3, it follows that each a is imaginary, and we have no real curve.

A second attempt to fit this five constant equation to the data for the fifteen counties was made, taking the five points at fifteen year intervals from 1860 to 1920. This time, all roots of k were less than 2.4 millions, while the 1905 population, y_3, was 2.6 millions. Thus, again, there results no real curve.

Fitted to the data for the twelve counties, however, for the five points at twenty year intervals from 1840 to 1920, the curve is real, with the constants

$$\begin{aligned} k &= +3.887 \text{ millions} \\ a_0 &= +4.323 \\ a_1 &= -0.1469 \\ a_2 &= +0.002862 \\ a_3 &= -0.00002834 \end{aligned}$$

and the form shown in Chart VIII.

A comparison of the results of fitting the data for the twelve counties by different methods, making allowance in the usual way for different

[1] Pearl, *loc. cit.*, p. 577. $a_1 = \frac{18\beta_1 - 9\beta_2 + 2\beta_3}{6x_1}$,

$$a_2 = \frac{4\beta_2 - 5\beta_1 - \beta_3}{2x_1^2},$$

$$a_3 = \frac{\beta_3 + 3\beta_1 - 3\beta_2}{6x_1^3}.$$

numbers of parameters, n_c, in different equations, is made in Table IV. In this table, $n=9$, and $n_c=$ the number of constants in the equation.

TABLE IV

Curve	Upper asymptote (in millions)	$\frac{\Sigma(\Delta_y)^2}{n}$	n_c	$\frac{\Sigma(\Delta_y)^2}{n-n_c}$
Three constant				
Table I, line M	3.783	0.0272	3	0.0408
N	3.980	0.0213	3	0.0319
J	5.690	0.0020	3	0.0030
Table II, line J_2	5.474	0.0015	3	0.0023
Two cycle	6.334	0.0011	6*	0.0032
Two cycle	6.334	0.0011	7	0.0048
Five constant	3.887	0.0132	5	0.0297

* The two cycle curve depends upon seven constants, three for the lower cycle, and four for the upper. But $d=0$ for the upper cycle for this curve, consequently the results for $n_c=6$ as well as for $n_c=7$ are shown.

APPENDIX III

DETERMINATION OF THE CONSTANTS OF THE FOUR CONSTANT CURVE BY ALGEBRAIC SOLUTION

Since Pearl fits the three constant curve to three, and the five constant to five points, equidistant in time, it seemed of interest to try this same procedure with the four constant curve, fitted to the points $(0, y_0)$, (x_1, y_1), $(2x_1, y_2)$, and $(3x_1, y_3)$.

Writing equation (3) in the form

$$y=d+\frac{k}{1+me^{a_1x}}, \tag{3a}$$

and putting

$$\begin{cases} y_1-y_0=\theta_1 \\ y_2-y_1=\theta_2 \\ y_3-y_2=\theta_3, \end{cases}$$

and

$$e^{a_1x_1}=p,$$

(from which it follows that $e^{2a_1x_1}=p^2$, $e^{3a_1x_1}=p^3$), we find (on substituting the above coördinates for x and y in (3a) and solving the four resulting simultaneous equations for p, d, m, and k) p given by the quadratic expression

$$p^2-p\left[\frac{\theta_2}{\theta_3}+\frac{\theta_2}{\theta_1}+\frac{\theta_2^2}{\theta_1\theta_3}-1\right]+1=0 \tag{21}$$

(a root $p=1$, corresponding to $x=0$, having been discarded). This double value for p results in two sets of constants, d, k, m, and a_1, since their values depend on p as follows:

$$d=\theta_2\frac{(y_1p-y_0)-\theta_1p(y_2p-y_1)}{(p-1)(\theta_2-\theta_1p)},\ \text{(where } p\neq 1\text{)} \tag{22}$$

$$m=\frac{\theta_1}{(y_0-d)-p(y_1-d)},\ \text{or } a_0=\log_e m \tag{23}$$

$$k=(y_0-d)(1+m) \tag{24}$$

$$a_1=\frac{1}{x_1}\log_e p. \tag{25}$$

The results of several trial fittings of this curve are shown in Table V. No test of the fit of these curves was made.

TABLE V

Trial	Data	p	a_1 $\left(=\frac{1}{x_1}\log_e p\right)$	m	a_0 $(=\log_e m)$	k (in millions)	d (in millions)	Lower asymptote (in millions)	Upper asymptote (in millions)
	Fifteen counties:								
I (a)	1890–1920, $x_1=10$.....	Imaginary	.....	.....	.....	.....	.....	.....	.....
(b)	.....	Imaginary	.....	.....	.....	.....	.....	.....	.....
II (a)	1875–1920, $x_1=15$.....	1.8918	+0.0425	+0.184	−1.693	−7.699	+7.287	−0.412	+7.287
(b)	.....	0.5286	−0.0425	+5.429	+1.692	+7.695	−0.412	−0.412	+7.283
	Twelve counties:								
III (a)	1890–1920, $x_1=10$.....	Imaginary	.....	.....	.....	.....	.....	.....	.....
(b)	.....	Imaginary	.....	.....	.....	.....	.....	.....	.....
IV (a)	1860–1920, $x_1=20$.....	4.3510	+0.0735	+0.0426	−3.156	−4.655	+4.818	+0.163	+4.818
(b)	.....	0.2298	−0.0735	+23.48	+3.156	+4.653	+0.163	+0.163	+4.816

The data of this region sometimes give imaginary values for the constants of the four and five constant equations, but this does not happen when the first differences of the population series used always increase with x.

The calculation of the four constants by the algebraic method is not difficult, and the two resulting sets of constants give the same curve. It should be noticed (from Table V) that when k is negative, the signs of a_0 and a_1 are merely the negatives of their values when k is positive. That is to say, the right-hand member of the equation

$$y=d+\frac{k}{1+e^{a_0+a_1x}} \tag{3}$$

has two terms, the first of which is a constant, and the second a variable. In trials II (a) and IV (a) of Table V this variable term becomes $\frac{-k}{1+e^{-a_0+a_1x}}$ as contrasted with $\frac{+k}{1+e^{+a_0-a_1x}}$ for II (b) and IV (b), if we agree for the moment to mean by k, a_0, and a_1 their absolute values. Now, as x increases, the exponent of $e\left(\text{in } \frac{-k}{1+e^{-a_0+a_1x}}\right)$ increases, and the value of the fraction approaches zero; that is, the second term of

(3) becomes a smaller and smaller negative value, ultimately approaching zero, and the upper asymptote of the curve is therefore d. If x takes on larger and larger *negative* values, the value of the fraction above approaches $-k$, and hence the lower asymptote of the curve is given by $y=d-k$.

The constants of trials II (b) and IV (b) Table V are the constants in their usual form and need no further comment, except perhaps to notice the possibility of the method giving a negative d, *i. e.*, a negative lower asymptote, as in trial II.[1] This, of course, appears to violate Pearl's assumption that the lower limit of population is zero, and cannot be negative since a negative population is an impossible concept. A possible interpretation may be as follows:

Imagine two symmetrical logistics, one of which has a smaller range between its asymptotes than the other. If the second logistic is superposed on the first so that their points of inflection are fairly close, the curves might be interpreted as representing a hypothetical population growth, the first representing a lower cycle, the second, an upper cycle. If the lower asymptote of the lower cycle is small, or zero, the upper cycle would necessarily have a negative lower asymptote, but there would not arise any occasion for imagining a negative population. The two cycle curve would merely represent a population growth for which conditions changed during the (approximately) straight portion of the lower cycle, the change not being violent or sudden, and operating chiefly in lengthening and changing the slope of this "straight" portion, in the shift to the upper part of the second cycle.

APPENDIX IV

POPULATION DATA USED IN FITTING THE LOGISTIC CURVE FOR THE CHICAGO REGION

Date of U. S. census	Fifteen counties		Twelve counties	
	Population (in millions) reported by the census	Population (in millions) estimated as of January 1 by logarithmic interpolation	Population (in millions) reported by the census	Population (in millions) estimated as of January 1 by logarithmic interpolation
1840 June 1	.....	.....	0.053516	0.050879
1850 June 1	.....	.....	0.180224	0.171399
1860 June 1	0.402383	0.394070	0.363517	0.353058
1870 June 1	0.664422	0.650786	0.612733	0.599648
1880 June 1	0.960136	0.945540	0.905274	0.890690
1890 June 1	1.609844	1.575983	1.547982	1.514024
1900 June 1	2.349570	2.313124	2.276813	2.240780
1910 April 15	3.047324	3.024556	2.971633	2.948677
1920 January 1	3.858818	3.858818	3.785224	3.785224

[1] In this particular case the negative value may not differ enough from zero to be statistically significant, but the possibility of a significant negative value for other data is suggested.

RECENT IMPROVEMENTS IN GERMAN SOCIAL STATISTICS

By Mollie Ray Carroll, *Berlin, Germany*

The improvement in German industrial life following the War and inflation is bringing with it some important refinements in social statistics. Of importance from this standpoint is the census of 1925, the first since 1907. In addition, however, statistics of wages and of cost of living have been improved over those of the pre-war period. Advances have been made not merely through the resumption of the collection of data that were not gathered during the War, but also through the collection of new material and the application of improved methods.

The task of German statisticians in these fields has not been easy. The industrial and economic life of Germany has been disrupted and altered to a degree scarcely understandable by those who have not seen the effects of the War, the writing of a democratic constitution in 1918, inflation, and the movement toward "rationalization" of industry which have all contributed to the marked changes in German economic life.[1] Any one of these phenomena alone would have been sufficient to cause enormous changes in economic and social conditions. The four of them, compressed into the period of about a dozen years, have resulted in profound modifications. Statisticians have, therefore, faced the need of developing measurements for a changed and changing order. They are trying to work out statistics that will be of value for future comparisons. Their purpose has been mainly either the pragmatic one of trying to arrive at an understanding of prevailing conditions or the attempt to secure some degree of statistical continuity throughout the difficult period from 1913 to the present. For the future this procedure is wise, but it undoubtedly limits comparisons that may be made with the past.

The last German pre-war investigation of family income and expend-

[1] A fascinating book on the Weimar Constitution of 1918 is Stier-Somlo, *Die Verfassung des Deutschen Reichs vom 11 August 1919. Ein systematischer Ueberblick.* A. Marcus and E. Webers Verlag. Bonn. 1920. The story of inflation may be found in Kuczynski, R.: *Post-War Labor Conditions in Germany*, U. S. Bureau of Labor Statistics Bulletin, 380. The book covers more than the inflation period, but the account of the inflation is excellently given. A book usually accepted in Germany as authoritative on the movement for rationalization of industry is Gottl-Ottlilienfeld, *Wirtschaft und Technik* in *Grundriss der Sozialökonomik*. In explanation it may here be said that rationalization is somewhat like our scientific management movement. It has, perhaps, laid most emphasis upon the technique of production on the one hand, and on national planning and organization of production and consumption on the other. Further, it has seemed to compress into one year the reorganization of the technique of production which we experienced over a series of years.

itures was for the year, 1907–08.[1] This study was based upon family budgets collected from 852 poorly circumstanced households that had kept these budgets for periods ranging from four months to one year. There is no discussion of the term, "poorly circumstanced," nor is there any description of the families, 36 of whom seem to have been families of salaried employees. The average income of these families was given as 2,192.08 marks yearly, and the average expenditure as 2,234.02 marks. This meant an average excess of expenditures over income of 41.94 marks. Budgets for "normal" families of parents with young children were gathered from 391 of these 852 households. Here the average expenditure was 2,110.38 marks.

On the basis of these budgets percentages were made showing the proportionate expenditures for food, clothing, rent and household requirements, heat and lighting and miscellaneous needs. The difficulty with these percentages and the estimates for use in comparison with present-day working class standards is that the actual wage rates of many workers today are less than the total called for. Of the eleven wage classes, for example, established by the unemployment insurance law of 1927,[2] seven fall below the average income of this 1907–08 budgetary study. Of the recipients of unemployment relief on July 2, 1926, fully two-thirds, and of the recipients of unemployment benefits on February 15, 1928, 64.7 per cent received weekly wages that would have given them less than the 1907–08 average, if they had worked full time throughout the year. Further, this discrepancy would be greatly increased if changes in the purchasing power of the mark between 1907–08 and the present time were taken into account. These figures, therefore, lead one to suspect that these 1907–08 budgets were collected not from poorer laboring class families but from those better circumstanced. One reason for the selection of the higher salaried households is that the requirements for keeping a budget for a year tend almost automatically toward the selection of persons or families belonging to the upper wage or salaried levels.

During the period from 1919 to 1925 an attempt was made by the Federal Statistical Office to reorganize the index of cost of living and the percentages of the budget required for food, clothing, housing, etc. The result was a new index of cost of living, first tried out in 1922,[3] which sought to calculate the needs of a poorly circumstanced family over a period of four weeks. The family was assumed to consist of two

[1] *Erhebung von Wirtschafterechnung minderbemittelter Familien im Deutschen Reich*, Bearbeitet im Kaiserlichen Statistischen Amt. Sonderheft zum Reichsarbeitsblatt, 1909.

[2] §105 of the law.

[3] *Vierteljahreshefte zur Statistik des Deutschen Reichs*, 1920, pp. 158 ff.; *Wirtschaft und Statistik*, 1921. p. 322.

adults, a boy of fourteen, a girl of seven, and a child of one and one-half years. Allowance was made for a certain amount of quantity buying. An attempt was made to work out a commodity budget for a working class family. This involved giving different weights to various items than had been used in the budget of 1909. Costs and proportions of expenditures for various items were calculated from prices collected in over 40 localities in different parts of Germany.

This study of 1922 was designed to serve at least two purposes. It was to aid in better calculation of proportionate expenditures for various items of the budget and for future reckoning of changes in cost of living. For this purpose it was very definitely affected by the necessities of the inflation period. It was based on the most essential and cheapest requirements for a bare existence. In the second place, the new index was used to compare current cost of living with that of 1913. In this attempt it encountered two difficulties. In the first place, consumption standards were undoubtedly very different and much lower during the inflation period than in 1913. The base used for 1913 was, however, not the actual consumption during that year, but the cost of the budget constructed for 1919–22. Actual comparison, therefore, of the standard of living of the people during these two periods cannot be made on the basis of the index of 1922.

In the second place, price data for 1913, on the basis of which the index was constructed, were obtained retrospectively at the time of the study of 1919 by assembling available price data for 1913. The limitations of data collected several years afterwards and particularly after an interval so thoroughly disrupted by war may be appreciated.

The final result of efforts from 1919 to 1925 to reorganize the cost of living index resulted in the appearance of a new index in 1925. This was peculiarly necessary because a preliminary index number for 1922 was markedly influenced by the inflation when consumption standards were below normal for all classes of the population. Again the index number for 1925 was based upon a commodity budget, different from that for 1919, but also different from that for 1907. It took into consideration the depression in standards of living resulting from war and inflation, but was less affected by the actual period of inflation itself than was the index number for 1922. An attempt was made to reorganize the food budget. The figures representing actual consumption in 1907 were replaced by estimates on a diet including minimum food requirements in 1924. Calculations, as in 1919, were made on the basis of the necessities for a family of five for four weeks. Again they were given in proportions of the total budget required for various items. Provision was made for more meat, fats, eggs, and milk

than in the budget of 1919. Allowance for car fare was increased over 1907 because the intense shortage of houses necessitated much larger expenditure for transportation in the average working class family. Further refinements in attempts to calculate the cost of clothing characterized the reorganization of the index in 1925. Prices were collected from 72 localities in all parts of Germany, representing large, medium-sized and small cities, industrial and agricultural districts. In addition, an effort was made to secure more accurate data concerning prices in 1913.[1] From the standpoint of a rational basis for estimating the cost of living for the future, this effort was exceedingly valuable. From the point of view of comparison with pre-war conditions it could have but limited application. The figures for 1913 which were used as the base for the index number were recognized as quite faulty. The year was not a "normal" one, nor were the statistics adequate to give a picture of the actual situation in that year. The purpose in choosing data for 1913 was that of securing a hypothetical base, as little affected by the War and inflation as possible, and yet as near as practicable to current conditions, for each year's lapse increased the inaccuracy of the data.

Results of the study of 1925 were given in percentages of the total budget required by a poorly circumstanced family for various items. Comparisons of the proportions of 1909 and 1925 are significant and are as follows:

Items of the budget	Proportions to be expended according to the estimates of	
	1909	1925
Food	45.5	54.77
Housing	18.0	20.35
Heat and light	4.1	5.55
Clothing	12.6	10.05
Transportation	19.8	3.07
Miscellaneous		6.21
Total	100.00	100.00

The indices of these separate items for 1925, using 1913 as a base were:

Food	145.3
Housing	71.5
Heat and light	138.0
Clothing	172.4
Miscellaneous, including transportation	177.1
Total	135.6
Total, exclusive of housing	151.9

[1] For discussion of this whole investigation see: *Vierteljahreshefte zur Statistik des Deutschen Reichs*, 1926, pp. 24 ff.; *Wirtschaft und Statistik*, 1925, pp. 159–163; *Statistisches Jahrbuch für das Deutsche Reich*, 1927, p. 297.

The cost of living, according to the index of 1925, was 135.6. Had the preliminary method of 1922 been used for computation, the index would have been 125.1. The difference is due partly to the better collection of data in 1925, and even more to the influence of very low standards of living of the inflation period.

As showing the comparison between post- and pre-war budgets, a significant item is the proportion allowed for housing. While other post-war prices were in general above those of 1913, rents were kept down by legal regulation.[1] Nevertheless, the proportion of the budget allotted to rents in 1925 exhibits an increase over 1909. This shows that the studies of 1909 and 1925 were based upon entirely different income or budgetary analyses; whether of similar or different groups, it is difficult to say. The other important item of the budget which shows change is that for miscellaneous needs. This includes all provisions for cultural purposes. The proportion allotted to this item has been challenged, quite irrespective of the budget of 1909, as being far too small. Several unofficial budgets made by groups or localities, notably those of Hamburg and of salaried employees, have given weight to the opposition to the official proportions[2] in case of these

[1] This index of 71.5 is itself significant of changes of standard of living. Through legal regulation, rents of pre-war houses have been kept down to a maximum of one-fifth more than they were before the War. Rentals of houses built after the War may be somewhat higher. An index of 71.5 rather than 120 or more is probably indicative of the more crowded post-war living conditions, arising, if from nothing else, from the intense shortage of houses.

[2] Family budgets collected in 1926 by the salaried employees from 43 families among their membership, and from the city of Hamburg from 80 families in 1925 and from 300 families in 1926, give the following proportions:

	Salaried employees	Hamburg	
		1925	1926
Food	36.5	45.04	35.16
Housing	14.3	11.95	10.43
Clothing	12.3	12.91	8.98
Heat and light	3.3	4.26	4.18
Miscellaneous	33.6	25.84	40.25

The Hamburg study of 1926 further gave proportions according to income groups:

	Income groups, Hamburg, 1926, in Rentenmarks								
	Under 2500 M	2501–3000	3001–3500	3501–4000	4001–4500	4501–5000	5001–6000	6001–7000	over 7000
Food	46.46	43.85	42.31	43.76	36.63	33.48	31.07	30.25	27.65
Housing	13.64	11.52	11.42	10.68	10.15	10.58	9.22	10.26	9.03
Clothing	7.50	7.78	7.59	7.95	9.31	9.36	9.82	10.10	9.95
Heat and light	5.03	4.52	4.48	4.05	4.03	4.62	3.99	3.89	3.78
Miscellaneous	27.37	33.33	34.20	33.56	39.88	41.96	45.90	45.50	49.59

miscellaneous needs. Since the index for miscellaneous items has increased more than that for any other, an increase or decrease in the proportion allotted to this group of commodities definitely affects the final index of cost of living. Furthermore, the official index is based upon the standards of poorly circumstanced, laboring class families. For those with higher levels of consumption, the proportion expended for miscellaneous items undoubtedly increases. Therefore, this study has definite limitations for estimation of possible changes in standards of living of the entire population of Germany. At the present time another budgetary study is in the making by the Federal Statistical Office in coöperation with the Department of Health. Budgets, kept for a year, were collected in 1928, and are now in the process of analysis.[1]

These figures are not entirely comparable with the federal official statistics, as, contrary to the federal figures, they include payments for taxes and insurance in the miscellaneous groups, while the official study entirely omitted both taxes and insurance. The salaried employees' data have been criticized on several grounds. The average income of the 43 families was high, 4055.88 marks yearly. The Hamburg study serves to show the increasing expenditure for miscellaneous items with the increasing income of the group.

Statistische Mitteilungen über den hamburgischen Staat, No. 20, *Die Lebenshaltung minderbemittleter Familien in Hamburg im Jahre, 1925.*

Aus Hamburgs Verwaltung und Wirtschaft. Monatsschrift des Statistischen Landesamts. 1 Juli, 1927, *Die Lebenshaltung minderbemittleter Familien in Hamburg im Jahre, 1926.*

Das Allgemeine Freie Angestelltenbund: *Die Lebenshaltung des Angestellten*, Berlin, 1928.

[1] The relation between actual wages and actual prices at any one time are difficult to secure, particularly since the official data are given in percentages of the total budget. One private study of a Rhenish city has been made, however, by Dr. Ludwig Fleischer, assistant in the Hygienischen Institut der Medizinischen Akademie, Düsseldorf. It is entitled "Ueber die Lebenshaltungskosten städtischer Arbeiterfamilien in Deutschland," and was published in *Zeitschrift für Hygiene und Infektionskrankheiten*, 104 Band. 3 Heft. (Berlin, 1925. Julius Springer Verlag). The study was made for September, 1924. It contains a careful analysis of food requirements for families of different sizes for a period of four weeks, with the prices of the commodities according to market prices for September, 1924. Rent, clothing, heat and light and washing and cleaning are also analyzed and priced. No estimates are made of miscellaneous items.

Costs are estimated for five different family groups as follows: I. First family group, consisting of a married couple; II. Second family group, married couple and one 13-year-old boy; III. Third family group, married couple, a 10-year-old boy and a 6-year-old girl; IV. Fourth family group, married couple, and two boys, aged 13 and 11; V. Fifth family group, married couple, two boys, aged 13 and 9, and an 11-year-old girl.

For these families the following costs in Rentenmarks have been worked out:

	Type of family				
	I	II	III	IV	V
Food	53.44	73.00	83.56	89.52	110.68
Rent	16.00	16.00	24.00	24.00	24.00
Heat and light	10.00	10.00	10.00	10.00	10.00
Washing and cleaning	5.00	5.50	6.00	6.00	6.00
Clothing	15.00	20.00	25.00	25.00	30.00
Total	99.44	124.50	148.56	154.52	181.18

Comparison of these needs was made with the wages of public service employees (of that period and city) who were in receipt of a supplementary family wage with an additional sum for each child. Four

Similar problems and refinements of statistical methods are encountered in information concerning wages. German wage statistics were not officially collected before the War. When official information concerning them was desired after the War, it was obtained by securing wage data reported for the pre-war period by trade unions and employers' associations, and using only the figures which both sides would accept. In this case, as in the cost of living study, it must be emphasized that 1913 was not chosen as a "normal" year or because the data were adequate, but as a more or less hypothetical base which served the purpose of showing current fluctuations. Official post-war statistics, on the other hand, are made from official reports of legally recognized and enforced trade agreements.[1] These official statistics, however, though made from reports covering wide areas and a great variety of industries and occupations, have had certain distinct limitations. They have given the results, not in wage classes but in averages, only, for skilled and unskilled workers. Further, they have represented not the actual wages received, but instead the rates set by trade agreements.

Up to the present time, because the wage data have been given in averages for skilled and unskilled workers, it has been very difficult to discover the range of wages for skilled or for unskilled or the actual rates received by the lowest or highest income groups. Some light on this problem was thrown by a study made as of July 2, 1926, for the pur-

wage groups were found, as follows: I. Unskilled workers; II. Semi-skilled workers without special responsibility; III. Semi-skilled workers with special responsibilities; IV. Skilled handworkers. Their wages varied on the basis of an 8- or 9-hour day. Their wages in Rentenmarks were as follows:

Wage group	Unmarried workers	Married workers			
		Without children	With one child	With two children	With three children
I.					
8 hours	91.00	96.48	108.76	108.80	114.96
9 hours	101.92	108.48	115.36	122.16	129.08
II.					
8 hours	97.34	103.16	109.28	115.52	122.48
9 hours	109.08	115.80	122.72	129.68	136.64
III.					
8 hours	102.16	108.12	114.28	120.52	126.84
9 hours	114.48	121.20	128.12	135.36	142.24
IV.					
8 hours	113.48	119.48	125.80	132.04	138.26
9 hours	127.24	134.16	141.08	148.32	155.64
Estimated cost of bare necessities	.....	99.44	124.50	148.56 or 154.52	181.18

[1] These trade agreements were legally recognized by order of 1918, *Verordnung über Tarifverträge, Arbeiter-und Angestelltenausschüsse und Schlichtung von Arbeitsstreitigkeiten vom 23 Dez., 1918;* and by the industrial courts law of 1926, *Arbeitsgerichtgesetz vom 23 Dez., 1926.*

pose of establishing wage classes to aid in determining rates for the forthcoming unemployment insurance law. For persons unemployed on that day statistics were gathered of the last wage rates received previous to their unemployment. These statistics were secured from questionnaires sent to employers and covered 1,594,300 of the 1,740,754 persons who were on that day in receipt of unemployment relief. Returns for the remaining persons were not forthcoming or were, for various reasons, unusable.

There were certain statistical limitations to this study as a picture of wage rates as a whole in Germany. They were designed only to aid in establishing unemployment insurance classifications and rates. They were data as to earnings of unemployed presons, and therefore, perhaps, included a larger proportion of unadjusted workers than was to be found in the general, employed population. They probably represented a higher percentage of persons in the lower income groups than usually obtained. On the other hand, the inquiry took place in the middle of summer when the lowest paid, agricultural workers were practically all employed. A further difficulty arose from the fact that the period of receipt of last wages varied from a few days to six, twelve, or even more months previous to July 2, 1926; so that possible differences in wage rates arising from seasonal and cyclical changes may have appeared. This was particularly likely to be the case since 1925 had been a favorable and the first of 1926 an extremely depressed period in the business cycle.

On the basis of the study of July 2, 1926, wage classifications were

TABLE I

WAGE CLASSIFICATIONS MADE BY THE UNEMPLOYMENT INSURANCE LAW OF 1927 AND PER CENT OF PERSONS IN RECEIPT OF UNEMPLOYMENT INSURANCE BENEFITS ON FEBRUARY 15, 1928, WHOSE AVERAGE WAGES FOR THE THREE MONTHS PREVIOUS TO LOSS OF WORK FELL WITHIN THE VARIOUS WAGE CLASSES

Wage class (in Rentenmarks)	Recipients of maximum rates of benefit in percentages					
	Male		Female		Total	
	Actual	Cumulative	Actual	Cumulative	Actual	Cumulative
I (10 and under)	0.8	0.8	7.6	7.6	1.7	1.7
II (over 10 to 14)	1.4	2.2	14.0	21.6	3.1	4.8
III (over 14 to 18)	2.7	4.9	19.6	41.2	4.9	9.7
IV (over 18 to 24)	7.6	12.5	25.7	66.9	10.0	19.7
V (over 24 to 30)	13.4	25.9	16.9	83.8	13.8	33.5
VI (over 30 to 36)	17.2	43.1	8.7	92.5	16.1	49.6
VII (over 36 to 42)	16.8	59.9	3.9	96.4	15.1	64.7
VIII (over 42 to 48)	15.1	75.0	1.7	98.1	13.3	78.0
IX (over 48 to 54)	10.5	85.5	0.9	99.0	9.3	87.3
X (over 54 to 60)	7.0	92.5	0.5	99.5	6.1	93.4
XI (over 60)	7.5	100.0	0.5	100.0	6.6	100.0

established for the operation of the unemployment insurance law of July 16, 1927. The necessities of administration promise to give continuous statistical data of the average wage earned during the three months of employment just previous to the loss of work by the person who qualifies for unemployment benefits. Table I shows the wage classification of the unemployment insurance law of 1927 and the average wages—for the previous three months employed—of those in receipt of benefits on February 15, 1928.

The total number in receipt of maximum benefits on that day was 1,220,047, of whom 1,060,893 were males and 159,154 were females. Wage data are not at present similarly published for all who are compulsorily insured under the unemployment insurance law. This is largely because contributions to the fund are sent by the employers to the sick insurance fund which forwards them immediately to the unemployment insurance fund. Benefits, on the other hand, are paid to the unemployed persons by the employment offices directly charged with administration of the unemployment insurance law. It is quite natural that there should be more interest in unemployment statistics on the part of the organization erected to administer that law than on the part of the sick fund authorities.

The improvement in collection of statistics that has resulted in some new data on wage classification has arisen from administrative problems connected with the enforcement of the unemployment insurance law. The other type of progress is due to refinement of methods of gathering statistics on the part of the Federal Statistical Office. By the law of July 14, 1927, this office was empowered to collect more accurate wage data. Specifically, this was interpreted to mean the substitution of data on actual wages received for those of trade agreements (Tarifverträge). The Tarifverträge are established for an industry or occupation by joint agreement between the employer and trade union. If the two sides cannot come to terms, decision is made by the industrial court under its powers of compulsory arbitration. Further, the industrial court has power to extend the operation of a fairly widespread wage agreement to make it prevail throughout an entire territory or trade. In this way it may cover many unorganized as well as trade union workers. The Tarifverträge usually represent the minimum wage. In times of business prosperity or demand for a particular type of labor, actual wages may far exceed those set by the legal trade agreement. Therefore, statistics which give only Tarifverträge rates, though easy to secure because the rates are officially published by the industrial court, may be highly inaccurate.

By virtue of the power granted it by the law of July, 1927, the

Federal Statistical Office, aided by the Ministry of Labor, sought to remedy this weakness in German wage statistics by securing actual wage rates for about a dozen major industries, to publish results on one of each of these industries from time to time, and from year to year to have a basis of comparison of wages in each of these major groups. The first industry for which wage data were collected was the textile. The study was made in September, 1927.[1] Statistics were gathered for piece workers in 263 different establishments in 19 different localities and 11 branches of the textile industry. They covered over 35,000 workers, or about 21 per cent of the textile workers in those branches and localities listed in the 1925 census. They were based upon reports of the employers, who were required to coöperate, according to the terms of the law of July, 1927. The statistics sought to answer two questions, the relation of actual wages to trade agreement rates, and also the relation of present to pre-war wages.

Results of this study were given in averages and also in wage classes for male and female skilled workers and helpers respectively.[2] It was found that the average wage received by the skilled workers through trade union agreement exceeded the legal trade agreement rate by 26.6 per cent in the case of men and by 18.4 per cent in the case of women workers. In the case of helpers, the average hourly wage received exceeded the trade agreement rate by 10.3 per cent for men and 7.2 per cent for women. Certain limitations in the study, however, modified these results. The data were gathered in September, 1927. This was a period when the business cycle was at its crest. There was, therefore, good demand for labor. The working day was long and the

[1] See *Reichsarbeitsblatt*, No. 9, March, 1928; *Wirtschaft und Statistik*, No. 5, March, 1928.

[2] The percentages of persons in the textile industry in Germany in receipt of varying amounts of wages (according to the official investigation of September, 1927) are as follows:

Wage classification in Rentenmarks	Percentages receiving wage rates			
	Skilled workers		Helpers	
	Male	Female	Male	Female
To 20	2.2	6.4	2.3	24.4
Over 20 to 24	2.5	12.6	4.4	41.4
" 24 to 28	4.5	21.1	8.9	25.4
" 28 to 32	7.0	25.1	26.3	7.0
" 32 to 36	10.5	13.8	24.2	1.4
" 36 to 40	14.9	10.0	16.8	0.3
" 40 to 44	16.3	5.5	9.7	0.1
" 44 to 48	13.9	2.7	4.5	....
" 48 to 52	10.6	1.7	1.6	....
" 52 to 56	7.6	0.7	0.6	....
" 56 to 60	4.4	0.2	0.5	....
" 60	5.6	0.2	0.2	....
Total	100.0	100.0	100.0	100.0

wages were exceptionally high. Moreover, the study was made just previous to the expiration of old trade agreements in the textiles in many of the localities, and the discrepancy between trade agreement and actual wage rates was at its highest. The actual increase in trade agreement rates between September 1, 1927, and February 1, 1928, amounted to 9.5 and 10.8 per cent, respectively, for male and female skilled workers, and to 8.1 and 6.6 per cent, respectively, for male and female helpers, still leaving a wide margin of variation. The difference between actual and Tarifverträge rates, however, does not include the above-mentioned effects on weekly wages of the longer working day in September, due to good business conditions, but merely shows the hourly rates. Such comparison of trade agreement and actual wages is highly important, however, since it shows the inaccuracy of Tarifverträge data as a basis for knowledge of wages received and points to the necessity of continued collection of actual wage statistics. Whether or not this difference in actual and Tarifverträge rates in the textiles is indicative of the situation as regards other industries is today not known. Further studies of the Federal Statistical Office will, therefore, be of great importance.

In this investigation attempt was made, further, to compare present-day actual wages in the textiles with those of 1913. The actual hourly rates exceeded those of 1913. However, since there had been a decrease in number of hours worked during the week, the indices of weekly wages for September, 1927, were not so high as for daily wages. This was in spite of the fact that in September, 1927, the average working week for male and female skilled workers and helpers ranged from 49.5 to 53.1 hours, and overtime ranging from 2.8 to 6.1 hours weekly was reported. The normal working week of 1927 would, therefore, probably not have shown so high an increase of wages over 1913 as did the exceptional period of good business conditions. The indices of hourly and weekly wages for September, 1927, using 1913 as a base were as follows:

	Index of average wage received	
	Hourly	Weekly
Skilled male workers	189	165
Skilled female workers	199	173
Male helpers	184	170
Female helpers	180	158

The comparisons with 1913 also omitted the increased payments made by workers for taxes and for social insurance. The tax on wages amounted in 1927 in this industry on the average to 3.1 per cent for

male skilled workers, 2.1 per cent for female skilled workers, 1.8 per cent for male helpers, and 0.3 per cent for female helpers. Furthermore, where in 1913, contributions for social insurance were between 0.6 and 2.0 per cent of these workers' wages, in 1927 they amounted to between 6.9 and 8.3 per cent of the wages of workers in the textiles. This included, of course, only the portion paid by the worker and not the contribution of the employer. While the security granted by social insurance represented a real part of the worker's income, payments made for this security were a deduction from his weekly pay envelope. Another element requiring calculation in comparing wages of 1927 with those of 1913 was the change in the cost of living. The Statistical Office placed this at 150 for September, 1927. When account was taken of these factors of taxes, increased contribution to social insurance and change in the cost of living, the indices for these groups for September, 1927, stood as follows:

Skilled male workers	104.6
Skilled female workers	109.6
Male helpers	107.4
Female helpers	101.0

Even this final index is questioned. It is debatable whether comparison of wages or of cost of living between the present and 1913 can be made with any degree of accuracy. Since the figures for 1913 in case of both wages and cost of living have been made retrospectively, on the basis of data available after the War, their value is exceedingly limited for purposes of comparison. Frankly designed as an hypothetical base, they can hardly be interpreted otherwise. A further limitation of these data for purposes of comparisons between pre- and post-war conditions lies in the inadequacy of data concerning the comparative productivity of labor in the two periods. In the textiles rationalization has meant introduction of new machinery and methods, accompanied often by higher speed and production. Without a standard of comparison of productivity wage rates alone give an inadequate view of the situation. Finally, it must be borne in mind that the figures for the textiles may not be typical of trends in wages in other industries. Before the War the textiles in Germany were among the lowest paid industries. How far their progress represents the tendency in other industries may only be learned as wage data appear in the future.

Constant refinement of method is being sought in collection and use of social statistics in Germany today. The material published, however, cannot be rightly interpreted without understanding the difficulties and limitations of these statistics and particularly of the pre-war material which they use as a base.

ON THE ANALYSIS OF TIME SERIES

By Simon Kuznets, *National Bureau of Economic Research*

I. INFERENTIAL POSSIBILITIES OF THE ANALYSIS

Time series relating to economic data can be studied in two ways. We may treat them as purely historical records of some aspect of events that took place within specific, unrepeatable conditions of space and time. This is the way time series are used in economic history; for example, a series of output of the iron and steel industry in the United States during the years 1820–1920 may be taken as one document among a number of others relating to the same historical aggregate. In this use time series describe a single chain of events and form an item in the picture of an epoch which obviously will not repeat itself in its complex contents, the epoch, say, of American frontierdom and industrialization.

The other way to study time series is to segregate some components that would permit an inductive inference. In this case we are looking for some elements in the time series that have repeated themselves through a certain historical period and under varying conditions. The establishing of such a recurrence will permit us to make either a mechanical inference, that is an inference based upon a mechanical averaging of the repeated units, or will allow us, in a further study of the units thus gathered, to utilize them in an inference after welding them with congeries of some other data.

The unusual development of time series analysis witnessed during the last two decades was stimulated largely by the recognition that such inductive inference from time series is possible. This recognition is implicit in the assertion of the existence of cyclical fluctuations. For no matter how rough their regularity, or how easily the specific turning points of the economic cycles may be displaced by all kinds of random factors, the idea of recurrence, of repetition, is inherent in the idea of cyclical fluctuations. Once the existence of the latter was recognized, once it was seen that economic cycles in their repeated appearance produced repeatedly similar effects, the significance of isolating these phenomena from the composite picture of the unique historical epoch in which they were embedded became obvious. For where such isolation is carried through successfully, there opens up a possibility of generalizing the recurrence of these cyclical swings, and thus obtaining some basis for an inductive inference about the future.

The development of the whole mechanism of analysis became necessary since recurrent changes had to be separated from the non-recurrent ones, and later on, the recurrences of different amplitude and duration had to be distinguished one from another.

The significance of such an analysis is clear in the case of cyclical fluctuations. We segregate cycles as comparable, repeated units of one and the same phenomenon, and once this is granted, the possibilities of an inference have no logical qualification, but only an experimental one. In other words, we can go ahead and generalize, if the material shows no confounding diversity in the measures we choose as material for generalization. It will be admitted that if a considerable number of cycles is taken, and if due attention is paid to other data besides the statistical materials themselves, we can draw inferences, since we do count repeated appearances of phenomena of one and the same type.

The situation is altogether different in the field of correlation and secular movements. In correlation, if we compute a single index for a given historical period, there is no repetition of any kind. For we take into our scatter diagram all the pairs of items and fit one regression line to the composite array. We have no recurrences, for we take all pairs of items and throw them into the general group, without establishing whether or not each belongs to the same aggregate of historical condition as the other pairs. The line of regression that we obtain is thus a mechanical average of those single covariances, and as such a description of what happened during the given period of time, with that complex of events taken as a whole. Whether it is a good description has still to be tested by seeing how it compares with the values yielded by the different parts of the period in question. And its inferential value depends then upon its tendency to repeat itself or to change with the variation of historical conditions. Here again only this element can serve as material for an inference which shows a recurrence, and a single coefficient of correlation, just as a single cycle, is only a description of a unique, historical event.

The same reasoning applies to the trend line. In a line of secular movements, whether it be a moving average, a free hand or a mathematical curve, there are no recurring elements, or if present, they are irrelevant to the process of fitting. True, there may be what some economists call major cycles in the secular movements, and to that extent there is a repetition of one and the same course of movement. But unless such big cycles are established, the secular movement is essentially unrepeatable since it presents the course of events during a definite, historical period of time, and describes it as a *single unit*, as a *complete whole*. It is for this reason that some recurrent attributes

ON THE ANALYSIS OF TIME SERIES

By Simon Kuznets, *National Bureau of Economic Research*

I. INFERENTIAL POSSIBILITIES OF THE ANALYSIS

Time series relating to economic data can be studied in two ways. We may treat them as purely historical records of some aspect of events that took place within specific, unrepeatable conditions of space and time. This is the way time series are used in economic history; for example, a series of output of the iron and steel industry in the United States during the years 1820–1920 may be taken as one document among a number of others relating to the same historical aggregate. In this use time series describe a single chain of events and form an item in the picture of an epoch which obviously will not repeat itself in its complex contents, the epoch, say, of American frontierdom and industrialization.

The other way to study time series is to segregate some components that would permit an inductive inference. In this case we are looking for some elements in the time series that have repeated themselves through a certain historical period and under varying conditions. The establishing of such a recurrence will permit us to make either a mechanical inference, that is an inference based upon a mechanical averaging of the repeated units, or will allow us, in a further study of the units thus gathered, to utilize them in an inference after welding them with congeries of some other data.

The unusual development of time series analysis witnessed during the last two decades was stimulated largely by the recognition that such inductive inference from time series is possible. This recognition is implicit in the assertion of the existence of cyclical fluctuations. For no matter how rough their regularity, or how easily the specific turning points of the economic cycles may be displaced by all kinds of random factors, the idea of recurrence, of repetition, is inherent in the idea of cyclical fluctuations. Once the existence of the latter was recognized, once it was seen that economic cycles in their repeated appearance produced repeatedly similar effects, the significance of isolating these phenomena from the composite picture of the unique historical epoch in which they were embedded became obvious. For where such isolation is carried through successfully, there opens up a possibility of generalizing the recurrence of these cyclical swings, and thus obtaining some basis for an inductive inference about the future.

The development of the whole mechanism of analysis became necessary since recurrent changes had to be separated from the non-recurrent ones, and later on, the recurrences of different amplitude and duration had to be distinguished one from another.

The significance of such an analysis is clear in the case of cyclical fluctuations. We segregate cycles as comparable, repeated units of one and the same phenomenon, and once this is granted, the possibilities of an inference have no logical qualification, but only an experimental one. In other words, we can go ahead and generalize, if the material shows no confounding diversity in the measures we choose as material for generalization. It will be admitted that if a considerable number of cycles is taken, and if due attention is paid to other data besides the statistical materials themselves, we can draw inferences, since we do count repeated appearances of phenomena of one and the same type.

The situation is altogether different in the field of correlation and secular movements. In correlation, if we compute a single index for a given historical period, there is no repetition of any kind. For we take into our scatter diagram all the pairs of items and fit one regression line to the composite array. We have no recurrences, for we take all pairs of items and throw them into the general group, without establishing whether or not each belongs to the same aggregate of historical condition as the other pairs. The line of regression that we obtain is thus a mechanical average of those single covariances, and as such a description of what happened during the given period of time, with that complex of events taken as a whole. Whether it is a good description has still to be tested by seeing how it compares with the values yielded by the different parts of the period in question. And its inferential value depends then upon its tendency to repeat itself or to change with the variation of historical conditions. Here again only this element can serve as material for an inference which shows a recurrence, and a single coefficient of correlation, just as a single cycle, is only a description of a unique, historical event.

The same reasoning applies to the trend line. In a line of secular movements, whether it be a moving average, a free hand or a mathematical curve, there are no recurring elements, or if present, they are irrelevant to the process of fitting. True, there may be what some economists call major cycles in the secular movements, and to that extent there is a repetition of one and the same course of movement. But unless such big cycles are established, the secular movement is essentially unrepeatable since it presents the course of events during a definite, historical period of time, and describes it as a *single unit*, as a *complete whole*. It is for this reason that some recurrent attributes

of a mathematical line of trend (such as the constant increase of a straight line) are entirely irrelevant. We do not fit the successive increases of the trend line to the series, and thereby find the series repeatedly showing a constant increase. We fit the line as a whole, presenting a composite picture of the situation through the complete period covered by the series. Inferences from trend could be made only if we had repeated fitting to varying historical periods, or if we fit to varying branches of activity during different periods of time and space, and then find some common characteristics.

If, in spite of that, we have forecasting done from a single line of trend, from a description that is thus bound to be historically limited, the assumption is that the forces that have been determining such movement in the past, will continue to do so in the future, will repeat themselves. The basis of expectation here is not at all the statistical analysis, but that information from a different source which enables the forecaster to assert that the period for which the trend line was fitted was "homogeneous," that is, under a preponderant influence of one and the same known set of forces, which is expected to repeat its influence in the future. On the contrary, the logical basis of the statistical analysis is that the cycles are repeatable units, while the trend line summarizes all the continuous movements that show no recurrence.

This, however, suggests a rather peculiar difficulty. If the process of analyzing a time series into cycles and trends is essentially a decomposition of the complex historical event into recurrent changes and a movement which describes the other considerable changes that are specific to this period only, how can we be sure that one is not influenced by the other? A time series is and always remains a description of a definite historical epoch. It has elements that vary during rather a short period of time: Those are the random changes which we may hope to get rid of by some averaging of the obtained numerous cycles. But the presence of a trend line testifies to the fact that there has been in operation a set of forces that made for considerable changes through the period as a whole. Perfect as the mechanical isolation of the two graphically-observed components may be, can we be sure that the specific course of the trend line for the given period has not affected the measurable characteristics of our repeated cycles? The number of the latter is never too great to establish completely the typical character of the average measures obtained, and we cannot easily say whether or not the average measures derived for a set of cycles for a given period do not contain an element specific to that epoch, due to the fact that during the years covered we had this and not another movement of the line of secular trend.

This difficulty is the one we would expect logically. Indeed, if we are trying to segregate the repeatable element out of its historical setting, we can never succeed quite surely, unless either the repetition is perfect, or we are helped along by the knowledge of some other uniformities besides the one we are trying to isolate and count. Since our cyclical recurrences are far from being mechanically perfect, we have to ascertain the nature of secular movements, in order to see whether in them we can find some uniformities, both in characteristics of their course and in their influence on the cyclical fluctuations. The knowledge of such uniformities will greatly reduce the purely specific, purely historical element in our secular movements, and thus also in the sets of our cycles.

In studies up to the present the emphasis has been placed upon isolating the cyclical fluctuations and using them to establish some important inference as to lags, to duration, to amplitude and the like. The secular movements were determined and forgotten, as if they were not a part of the composite phenomenon described by the series and thus could have no influence on the recurring oscillations. It would seem necessary to pay more attention to the nature and influence of secular movement, for beside its contribution to the stock of uniformities, it might bring light also into the rather ambiguous state that exists at present in the technique of the analysis. The inferential possibilities of the results of our analysis might be greatly augmented thereby.

II. THE CRITERIA OF TREND-CYCLE ISOLATION

The brief discussion above has set up as a logical basis of the trend-cycle distinction the difference between the repeating element and the historically continuous, specific non-repeated movement. It must be made clear that this non-repeatability of the secular movement is on the scale of time only, not on the scale of space. We might find a similar course of secular movements repeated in various branches of economic activity. But within one and the same branch, for the same period there is no repetition: The trend line gives the general characteristics of the composite, non-recurrent movement.

This point warrants stressing, even at the risk of pressing the obvious. For the use of mathematical curves as trend lines has served to cast an aura of regularity and recurrence on the idea of secular movements. Against this notion two considerations may be emphasized. (1) As far as the absolute values of the trend lines are concerned, they obviously cannot be recurrent, since a reversal of the direction of the trend is possible only as a singular event. (2) As to the functions of the trend lines, which seem to be recurrent in cases of

mathematical curves, it can again be stated that they are irrelevant to the process of description and are only mechanical means of producing that whole line of movements which presents the picture of the secular changes for the period as a whole. The only condition under which we could call some characteristics of a line of trend recurrent is that it repeats itself within successive periods, without destroying the continuity of the movements. Thus if we break a period into three parts and find it possible to fit a straight line to each part, in such a way that the last item of the first period trend continues without a break into the first item of the second period trend line and so on through the whole period, then we have both the historically continuous movement and a repetition of a certain characteristic under diverse historical conditions. It is obvious that our usual trend curves are descriptions of only single, unrepeatable courses of movements.

If secular movements are thus continuous and non-recurrent, this continuity and non-recurrence are obviously relative to some scale of time, the scale being determined by the duration of cyclical swings. Changes in the latter will affect considerably the results of the procedure adopted for the determination of secular movements. The precision of our knowledge of these movements depends to a large extent upon the definiteness of our conception of cyclical fluctuations.

This dependence is not dissolved by any refined mathematical methods of curve fitting or smoothing. In the latter the period of smoothing must be determined. In the former the general character of the curve to be fitted has to be decided upon. In both groups of cases our knowledge of the actually-observed cyclical fluctuations, or as an alternative, our knowledge of secular movements, must play a prominent part.

Since our analysis of time series into the line of secular trend and the cyclical oscillations-deviations is nothing but a separation of two components, we can obtain definite criteria of the goodness of separation only if we fix one of the components by some definite quantitative characteristics. Such are given to us by our logical distinction. But the difficulty lies in application to actual data. If we could apply fully our logical conception of cycles, and thus declare them perfectly recurring, we would not only obtain a precise test of the success with which the analysis has been carried through, but we could also embody this logical distinction in the mechanism of analysis itself. This would give us an infallible method of separation, the results of which would need no testing.

But the actual roughness of the cyclical fluctuations in the data does not allow us to raise the logical distinction into a mathematical

criterion, and renders every mechanical method a distortion of the analysis. Neither do we know enough about the exact course of secular movements. In such conditions the criterion becomes indefinite and relative to the exactitude with which we can establish beforehand one of the two components.

There is no one simple way out of this difficulty. However, a consideration of the purposes of analysis will show us that the rigid assumptions of the different methods available for use have a different negative value varying with changes in the purpose of the procedure. And on the other hand, we are able to determine certain things beforehand about the component parts that we have to isolate, and thus establish on more definite grounds our final judgment as to the success of the operation. These considerations that might help to make the mechanisms of analysis less vague and more pliable and self-conscious will be taken up in the order mentioned.

III. THE PURPOSES OF TREND-CYCLE SEPARATION AND THEIR BEARING ON THE METHODS

The purposes for which analysis of time series is undertaken may be divided into two groups. In the first, the interest is in measuring cyclical fluctuations themselves, and the secular movement is determined only to be eliminated and forgotten. In the second, the long-time changes are described in order themselves to be studied and made use of further. To the first group belong all the investigations of correlation; to the second, the studies in forecasting, the comparisons of past changes, the attempts to generalize as to the course of long-time movements.

The purpose of analysis should influence the methods of determining secular changes. In the first group we are not interested in any particular form or shape of the latter, and the elimination is guided by the conception of cyclical fluctuations undisturbed by long-time factors. The methods adopted should be thus quite flexible. In the second group these methods are confined more rigidly by our interest in the secular movements themselves. The desire to forecast, to make comparisons, or to verify a hypothesis determines a particular description, one that would best satisfy the purpose at hand.

This difference is illustrated in the selection of the period for which secular movements are determined and of the method of determination. When the purpose is only to get rid of the long-time changes, any period of time will do, provided it is longer than a cycle and contains a number of complete cyclical swings. But when the trends are to be used for further purposes the period must be chosen carefully. If forecasting

is desired, the choice must permit the inclusion of all (and only of) those influences which are most probable to persist in the future. Where comparisons are to be made, the period to be covered is fixed beforehand. And if some tentative generalization as to the course of secular changes is to be tested, the period must be so chosen that the forces presumably operating could have exercised their influence undistorted by any other factors.

The method of description also should be selected with regard to the purpose of analysis. In cases where we want only to eliminate the long-time movements, the most flexible types of description are proper. Simple or weighted moving averages are probably best, for without making any limiting assumptions as to the nature of secular changes, the smoothing operations are based upon given characteristics of the cyclical fluctuations themselves. The defects of this method are due to the fact that cyclical swings present in specific time series do not always exhibit the characteristics assumed. But by varying the period and by smoothing a good part of the difficulties can be overcome, while the flexibility of the method assures a completeness of elimination of long-time changes that is seldom achieved by a mathematical curve. Such complete elimination is supremely important in correlation studies.

By virtue of its flexibility, the moving average can be used in the second group of investigations, but as a first step only. For the final description of secular movements must in all such cases be simpler, reveal uniformities more conspicuously than is done by the line of moving average. In forecasting we need a curve that allows extrapolation of most probable values. Comparisons call for a line that embodies most prominently the unit of comparison. And if an hypothesis is to be verified, the secular movements should be described by a curve that incorporates the assumptions tested. In all these cases a moving average can be only a first step toward the final description of long-time changes by a mathematically expressed curve.

The methods of fitting the latter (once the general equation and the number of constants are determined) are not greatly affected by the purpose at hand. But we must bear in mind the essential uncertainty of the whole process of separation, not to be overcome by mechanical methods of fitting. The method of least squares may save the investigator the troubles of decision involved in fitting to selected points, and may appear more objective in the sense that identical results will be reached by different investigators. But mechanical arbitrariness is no whit better for being mechanical, and the method of least squares does not assure satisfaction of two most obvious criteria of goodness

of fit—the balance and the minimizing of deviations from trend for each particular cycle.[1]

IV. THE TWO GROUPS OF SECULAR MOVEMENTS

These brief remarks upon the possibilities of different methods of description and isolation in relation to various possible purposes might help to determine the choice of procedure, but do not make our criteria of judgment as to the results any more definite. This latter can be accomplished only by making more definite one of the component parts—either the secular movements or the cyclical fluctuations.

Taking up the question of the nature of secular movements, an important technical fact must first be noted. If by the secular movements we understand the continuous changes underlying the cyclical fluctuations of the presently recognized duration of three to seven years, then these movements are really components of two groups of changes: of primary and secondary secular movements, or if one wants to call them differently, of secular movements proper and of major cycles. This distinction was found necessary by the author in the course of his investigations into the nature of secular movements, and to its validity testify the results of a number of other investigations, lately multiplying.[2]

This revelation of the composite nature of what we at present call secular movements, is of immediate technical significance in the analysis of time series. For it is obvious that, given major cycles, simple curves cannot adequately describe secular movements, unless rather short periods of time are taken, or unless the large secondary swings are cancelled in their effect by the opposite movement of the primary trend line, a combination unlikely to occur. A straight line can form only a rough approximation to the actual course of secular movements.

The second bearing is on the question of trend stability, a question of annoying importance where the analyst is concerned not only with a finished period of the past, but intends to exploit it for the interpretation of the present or of the future. In such cases the question whether it is probable that the established trend line will maintain itself in the future is affected by our knowledge that in the past there were two types of movements in the composite line of trend. Their combined

[1] Neither has the connection of the least squares procedure with the theory of probability any significance in the analysis of time series. See admirable early discussion by W. Lexis, *Zur Theorie der Massenerscheinungen in der Menschlichen Gesellschaft*, Freiburg, i. B., 1877, pp. 31–33.

[2] See W. C. Mitchell, *Business Cycles*, 1928, pp. 226–30. It should be added that one of the leading investigators of these questions, Professor Kondratiev, has recently published an extensive article on the same subject of long cycles. See *Archiv fur Socialwissengchaft and Socialpolitik*, v. 60, heft. 1, pp. 1–85.

effect during the period elapsed may be described by a line which within that past will give a true picture of the secular movements, but which really depicts a mechanical summary of two groups of forces of different tendencies, the combined result of which may change within the immediate future.

Thus, in statistical analyses in the past, only those forecasts of trend were successful, comparatively, which related to series with mild secondary secular movements. This was the case in population and in those branches of production or consumption where the inelasticity of demand or the stability of productive conditions has not allowed considerable fluctuations in the line of primary growth. On the contrary, in series where the primary trend is not considerable, such as prices or money volumes, forecasts were decidedly disappointing.

In the future any discussion of secular movements will have to take into account the presence of these two component parts. It is of immense importance in cases where the forecasting of the trend is of any significance. And in cases where "pure" cyclical fluctuations are to be established with high precision, the knowledge of the complex character of the secular movements should preclude the use of simple curves, of straight lines especially. But, where the purpose is to study the cyclical fluctuations proper, our knowledge of secular movements is too general to serve beyond a mere warning against over-simplified descriptions. There a greater precision of testing can be achieved by fixing more definitely the cyclical fluctuations themselves.

V. ON FIXING THE CYCLES IN THE SERIES BEFOREHAND

When we fit a trend line and judge the results by inspection, we are really fixing the exact turning points of the cycles in the series before the segregation of the two components has been made. Indeed, in deciding by inspection whether the fit is good or not, we keep before our eyes the observable cyclical swings and following the line of trend see in what way it crosses those oscillations, whether it passes below or above the average level of the cyclical swings, whether it shows the same direction. Then by some vague general impression we decide that the fit is good enough or not good enough.

There seems to be no reason why this procedure should not be rendered more definite by the recognition and acknowledgment of the process involved. Instead of keeping the whole chart before our eyes and obtaining a general impression, we can fix definitely the dates of the cycles, write them out and then pass our judgment upon the trend line for each cycle separately. The fixing of the cycles, where the seasonal element is small, is not difficult. Where the seasonal is present,

smoothing by a moving average, which anyway precedes in a number of cases the computation of the seasonal, will again enable us to fix the dates.

This fixing has two advantages. First, in this process we can utilize non-quantitative data. Of course, where the series shows a conspicuous cyclical swing, we can establish the dates and the existence of the cycle by inspection only. But where doubt arises, and the showing of the plotted series itself is not sufficient for decision, other records can and should be consulted. The annals of the described branch of activity, the annals of the business life of the country as a whole at that particular time can be consulted, and the information contained there can be used to decide whether we should expect a cycle in this series and thus use it at least as a unit of test. The usefulness of methods that would draw in other data into the mechanism of time series analysis cannot be doubted.

The other advantage is that our final judgment of the goodness of fit of the trend line becomes much more definite. By listing the cycles, after they have been fixed, and by characterizing the behavior of the trend line within each cycle by some mark, we get a much more particularized idea as to the success of the analysis. This is especially important for the later use of the results of separation. For in case we adopt some imperfect trend line, and none is quite perfect, we want to keep in mind which of the cycles are affected by this misfit, and might influence our successive coefficients of correlation or our averages.

In short, this fixing of cycles beforehand allows a more discrete use of the mechanism of analysis. And in cases where mathematical curves are fitted as trend lines, it also allows a rather exact comparative test of the goodness of fit. (See appendix to this article.)

VI. SUMMARY

The following tentative observations are submitted in summary of these somewhat scattered reflections on the logic and mechanics of time series analysis:

1. The inferential possibilities of time series analysis are contingent upon segregating from the specific historical the repeatable, recurrent element. Such on the scale of time (as distinct from the scale of space) are the cyclical fluctuations.

2. Trend lines and correlation coefficients become proper subjects of inferences only if they are repeated. As averages for single historical periods they are specific to the period which they cover.

3. The logical basis of the secular movements-cyclical fluctuations

is that the former are non-recurrent on the scale of time, while the latter are recurrent.

4. The rough character of the recurrence actually observed does not permit formulating the logical distinction into a mathematical criterion and embodying it in an infallible method of fitting. This makes testing of the actual procedure rather vague.

5. In view of that the purpose of the analysis should influence the choice of method.

6. Knowledge of the composite character of secular movements should warn against using too simple curves.

7. The criterion of judging the results can be made much more definite by fixing beforehand the cycles in the series.

8. The rough character of the recurrences observed and the complexity of the historical material presented by time series necessitate greater attention to non-quantitative data and to the non-cyclical element of the series and less reliance on the powers of our rather too rigid mechanism of statistical analysis.

APPENDIX

TESTING GOODNESS OF FIT OF A CURVE TO A TIME SERIES

This test is to be applied only when it is desirable to have more than a visual idea as to the goodness of fit. It can be applied only where a mathematical curve is fitted as a trend line, and where we have fixed our cycles beforehand to serve as units of testing. The essence of this procedure consists in comparing our trend line with straight lines fitted to each of the observed cycles. The operations involved are as follows:

1. Obtain the sum of the actual items of the series by halves of each cycle.

2. Get the equation of the curve and integrate it for the limits of x (time) indicated by the half periods of each cycle. It should be noted that nearly all trend curves are easily integrable (the straight line, parabolae on arithmetic or log scale, the Logistic and the Gompertz). We get then a table something as follows:

Cycle 1885–1888	Sum of the original items of the series	Integrals of the curve
(1) First half	2500	2400
(2) Second half	2700	2550
(3) Difference between lines (1) and (2)	+200	+150
(4) Sum of lines (1) and (2)	5200	4950

The entries on line (3) give the trend direction of the straight line computed from the data for each cycle and that computed from the curve. Of course, the curve is misrepresented, but since the cycle periods are short the difference between the linear approximation and the curvature of the trend cannot be significant. The entries on line (4) give us an idea as to whether or not the curve passes on the average level of the cycle (entries in columns 2 and 3 equal) above it (entry in column 3 larger than in 2) or below (entry in column 3 smaller).

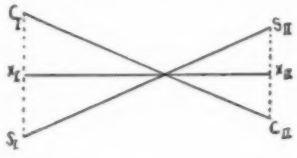
This, however, gives us two criteria of judgment instead of one. To get the one, we can easily compute the area difference of the curve from the successive straight lines computed for each cycle. In order to do that there should be computed:

1. The standing of the straight line from the data for the first and the last month of each cycle. This can be done by dividing the entries in column 2, lines (1), (2), by the number of months in the period, subtracting one-half of the difference from entry line (1), column 2, and adding one-half to line (2), column 2.

2. The standing of the straight line from the curve for the first or the last month of each cycle. This can be done by a process analogous to the one just described. Or the whole labor of integration can be saved, and the items computed directly from the equation of the curve. This, however, will give an error, the bigger, the greater is the curvature of the curve.

We shall then subtract these standings one from another, for the first month of the cycle and for the last, getting two differences. Calling them difference I and difference II respectively we get the following formulae for the area difference between the straight line of the series and the straight line of the curves.

Case I. Where difference I and difference II are of opposite signs:



S_IS_{II} = straight line from series;
C_IC_{II} = straight line from curve;
S_IC_I = difference I; $S_{II}C_{II}$ = difference II.

Then we have two similar triangles and
Area difference =

$$\tfrac{1}{2}\text{ difference I} \cdot X_IX_{II}\frac{\text{difference I}}{\text{difference I}+\text{difference II}}+$$

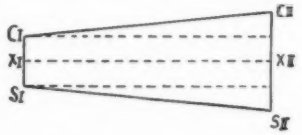
$$\tfrac{1}{2}\text{ difference II} \cdot X_IX_{II}\frac{\text{difference II}}{\text{difference I}+\text{difference II}}.$$

Here signs are disregarded.

Case II. Where difference I and difference II are of the same sign there are two possibilities.

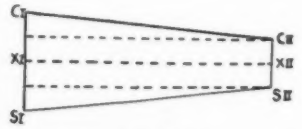
(a) Difference II is larger than difference I, and
Area difference

$$= \text{difference I} \cdot X_I X_{II} + \frac{(\text{difference II} - \text{difference I}) X_I X_{II}}{2};$$



(b) Difference I is larger than difference II, and
Area difference =

$$\text{difference II} \cdot X_I X_{II} + \frac{(\text{difference I} - \text{difference II}) X_I X_{II}}{2}.$$



In both cases we have a rectangle plus two or one rectangular triangles.

The sum total of these area differences can be taken as a comparative test of the goodness of fit.

It should be noticed, however, that while the method is not laborious, it is rather crude, involving the presentation of the curvilinear trend as a series of straight lines. In most of the cases inspection of the series specified by cycles fixed beforehand will suffice.

DEVICES FOR COMPUTING THE RATES TOGETHER WITH A STATISTICAL STUDY OF BUILDING AND LOAN ASSOCIATIONS

By C. C. Camp, *University of Nebraska* [1]

A common misconception prevalent even among bankers is the idea that in borrowing from a building and loan association advantage is taken of a low rate of interest, say around 5 per cent. There are two other reasons why such associations are popular in Illinois, where the 852 associations have nine million members. One is the fact that stock in them is non-taxable. The other is that human nature is such that necessary monthly or weekly payments for stock afford the only means by which many individuals can save. A well distributed sample from these 852 associations [2] shows that the average rate in that state is 8.87 per cent and that the average investor's rate in the same associations is 2 per cent less. This is legally not usurious, but since there are assets of $300,000,000 in Illinois associations and this is but 6 per cent of the total in the United States, it is certainly a matter of interest to know just what returns are paid on one's investment or what rate one is required to pay as a borrower. The difficulty appears when one attempts to determine mathematically these rates. One object of this paper is to compare possible methods, whether old or new, of computing these rates with a view to securing any desired accuracy with a minimum of numerical work. Another is to analyze some data calculated under the author's direction by W. W. Kriebel [3] from first-hand information obtained through a questionnaire sent out over the state in 1926. Since his results are as yet unpublished, certain items are here used by permission.

Whether the investor's rate or that of the borrower is sought there are two standard forms to which nearly all of the problems of building and loan associations can be reduced. This is not difficult to realize when one considers that except for variable payments at the beginning or end of the total time period all the payments are made at regular intervals of one month or one week. Although these two forms are reducible one to the other, it is practical to consider one as the type

[1] Presented to the American Mathematical Society, December 28, 1927, at Nashville, Tennessee.

[2] *Cf.* Thirty-fourth annual report on Mutual Building, Loan, and Homestead Associations, State of Illinois.

[3] A law student, University of Illinois, whose thesis on *The Borrower's Rate in Building and Loan Associations* was directed by the author.

form for computing the borrower's rate and the other for that of the investor. For the former one may use

$$Ma_m+Rv^m=P \tag{1}$$

where M represents the regular payment; P the amount of the loan at the beginning after all initial payments, including interest, regular dues, initiation, appraisal, and other entrance fees, have been deducted; and R is the residue exclusive of the regular payment which the borrower must pay at the end to extinguish the debt. This is usually $-M$. Sometimes it is not easy to find just when the stock will mature, but the book value may be available from a circular. In that case R is the difference between the book value and the maturity value of the stock; *i. e.* the amount then due on the loan, just after a regular payment. If $R=-M$ then $P=Ma_{m-1}$. One thus shortens the total period by one interval in order to avoid an annuity due.

For the investor's rate one may have an initiation fee, always small, and at the end no payment, part of a payment or a full payment of regular dues. In case the book value of the stock after a certain number of intervals is known the problem is no more complicated. In general one has the form

$$Fv^{-m}+Ds_m=S \tag{2}$$

where F is the initial payment including dues, D is the regular payment, and S is the book value of the stock at the end, adjusted for the last payment. If there is no entrance fee one writes $Ds_{m+1}=S$. Quite often no final payment of dues is needed. Then S is the maturity value of the stock plus D.

In both equations (1) and (2) $v=1/(1+j)$, where j is the interest rate per period and a_m, s_m are taken at rate j. The reason for using two forms is in order to have a relatively small term in each, namely Rv^m and Fv^{-m}.

A rare case appeared in which the form

$$70.3v^{132}+10.84a_{11}+10a_{120}\cdot v^{11}=975.41 \tag{3}$$

was encountered. One of the Danville associations charges a borrower 7 per cent for the first year and 6 per cent for the remaining time, each monthly in advance. The value of the stock on the basis of a thousand dollar loan was \$929.70 at the end of eleven years without final payment of interest or dues. Initiation fee \$1, dues \$5, interest at the beginning \$5.84, other fees \$12.75. A method for solving this will be deferred till later.

It is almost incredible that some textbooks use such a complicated

form as $(1+i)^{79/12}-(1+i)^{1/12}=99.60\ [(1+i)^{1/12}-1]$ when this may be expressed as $s_{79}=100.60$. The method of solution illustrated by it was Newton's, or the binomial expansion method. A mere glance is sufficient to show the cumbersomeness of this method in this case. The method consists of replacing the unknown variable i by i_1+h where i_1 is a first approximation. Only first powers of h are retained and the method to be made effective may require one or more repetitions depending on how close i_1 lies to the true value. In the absence of suitable tables the method may well be used in the simpler form of the above equation.

One of the simplest ways of solving this form is to use first differences in tables like Spitzer's, Glover's or Kent's new tables. Using Glover's one sees that the rate j lies between 7/12 per cent and 5/8 per cent. This gives $j=.005979$, whence $i=7.416$ per cent. The first application of the other method starting with $i_1=.072$ gives 7.42 per cent.

One might use second differences or a corrected first difference to get a closer approximation, but the following will be found of greater advantage since incidentally it checks the previous approximation: Take $1+j=1.00598$ to avoid interpolation in Schrön's Tafel I and find $s_{79}=(1.00598^{79}-1)/.00598=100.6048$. If this is not accurate enough use this together with the value of s_{79} for $j=.00583333$. By a second simple interpolation using a slide rule one obtains $j=.00597886$. With considerably more arithmetic using $j=.00598+h$ one may obtain $h=-.00000115$, *i. e.* practically the same result.

By the use of second differences one has $\Delta=1.7596$, $\Delta^2=.0393$. Then $100.60=99.9897+x[\Delta-\frac{1}{2}(1-x)\Delta^2]$, whence $x=.346$ approximately. Putting this value in the bracket one gets $x/2400=.6103/1.7467\times2400=.00014558$. Adding 7/12 per cent and cutting off one digit we obtain $j=.0059789$ which checks with the preceding value to five significant digits. The corresponding value of s_{79} checks to within a tenth of a mill. It is interesting to note that if the differences are of like sign the error in a function calculated by simple interpolation does not exceed 1/8 of the second difference, and in interpolation with second differences the error is less than .0642 times the third difference. This is obtained by putting the maximal value of x in the correction for the first unused difference. A similar rule may be stated for the case in which the differences are not of like sign but diminish by 90 per cent or more numerically each time. Then the corresponding errors are $\leqq(10/72)\Delta^2$ and $<\Delta^3/14$. By these rules one may determine the order of the error and the number of differences needed. There is no safer way to determine the actual error, however, than to test the final result in the original equation.

An extremely valuable method is that of interpolation with first or second differences in the logarithm of an annuity function from Glover's Tables. It is sufficient to take differences up to the third for log $s_{\overline{79}|}$ at 1/2, 7/12, 2/3 and 3/4 per cent to see how much more accurate this method is than when the natural values of $s_{\overline{79}|}$ are used. While the third difference in the logarithm is $-.0000007$, that for $s_{\overline{79}|}$ is .00076073. The maximum error in $s_{\overline{79}|}$ when only first differences are used in the logarithm is 4 mills, while for the ordinary method it is 2 cents. For this reason the majority of the rates listed in the statistical part of this paper have been worked out by simple interpolation in logarithmic functions. Actual checks showed an average error of less than ½ cent per hundred dollars in a group of 22 determinations. Interpolations with first differences in natural functions showed a considerably higher percentage of actual error.

What has been said about the simplest case of (2); *i. e.*, when F is zero, can be said analogously concerning the case of (1) in which $R=0$ or $-M$. For the last method mentioned a similar situation can be found for the differences in log a_m except that their signs alternate.

When the more general cases of (1) or (2) occur one approximates to the small term, Rv^m or Fv^{-m}, and applies the previous methods to the resulting forms. A second approximation to the value of these small terms is sufficient for ordinary accuracy when in (1) the book value is taken near maturity, since in (2) the initiation fee is always small. Of course these general forms can be solved by Newton's method, but other methods are less laborious unless rates are such that tables are not available for rates sufficiently close together.

If the total time involved is an integral number of years, forms (1) and (2) may be varied so that the effective rate may be determined directly. For instance, one might have such a form as

$$.25(1+i)^6+13s_{\overline{6}|} \cdot s_{\overline{1}|}^{(52)} \cdot (1+i)^{1/m}=90.45.$$

This is a weekly stock with an initiation fee of 25 cents, whose book value after six years is \$90.45. The first term may be ignored for the first approximation, also approximate values of $s_{\overline{1}|}^{(52)}$ and $(1+i)^{1/m}$ used. One may next approximate to the first term and use the logarithms from Glover's Tables for the resulting equation. A simple interpolation between 4½ and 5 per cent gives 4.798 per cent, which leads to an error of about 1 cent. The tremendous advantage of having the logarithms tabulated is quite apparent here.

Instead of approximating to the first term one may calculate the value of the whole left member of the given equation for the two rates and then interpolate in the natural numbers. The accuracy would

probably be less. This method may be called that of double false position. It may be employed in solving equation (3) above. A single interpolation between 1/2 and 7/12 per cent gives $j_1 = .0054985$. Putting this value in (3) one gets \$974.98 with an error of 43 cents. A second interpolation between this and the value for 7/12 per cent gives $j_2 = .0054905$. If one takes $j_2 = .00549$ to avoid interpolation, the value \$975.48 is obtained. By interpolating between these two checking values $j_3 = .54912$ per cent is easily obtained and hence $i = 6.792$ per cent.

Equation (3) may be solved by iteration if it is rewritten as

$$a_{120} = 97.541(1+j)^{11} - 7.03v^{121} - 1.084s_{11}. \tag{4}$$

Starting with $j_1 = 7/12$ per cent one gets $j_2 = .005385$. The error in successive approximations is reduced to about one-third of the previous error, so it will require seven operations to reach a result as accurate as j_3 above. Newton's method is also unsatisfactory here, since if one takes $j_1 = 7/12$ per cent, it leads to $j_2 = 0.005506$, with a large amount of arithmetic.

By accelerated iteration,[1] since $m_1 = -.3157$, one obtains $j_2 = .0054926$. A second application using .00549 gives $j_3 = .0054916$ and $j_3 = .00549 + .0000016/(1-m) = .0054912$, found above. It is not necessary to recalculate the value of m, but it is important to use second differences to get j_3 from log a_{120}. Since m_1 need be calculated only to three significant digits, it is apparent that this method gives maximum accuracy for the amount of numerical work.

The following data are used by permission from Kriebel's thesis:

	Investor's rate in per cent	Borrower's rate in per cent		Investor's rate in per cent	Borrower's rate in per cent
Kankakee	5.47	6.59	Altamont	7.27	8.35
American	6.00	6.60	Vermillion Co.	6.04	8.37
Hawthorne	6.53	6.73	Beardstown	7.92	8.89
Avondale	6.24	6.74	Carbondale	8.50	8.99
Danville	5.95	7.05	Concordia	9.74	9.23
Archer Avenue	5.78	7.10	Royal	6.38	9.32
Elgin	5.35	7.28	Central Illinois	10.30	10.14
Allied	6.50	7.35	Alton	7.80	10.21
Avenue	5.49	7.44	Waterloo	8.45	10.50
Silver Leaf	6.15	7.71	Black Hawk	6.92	10.90
Chesterfield	6.11	8.06	Arnoldsville	4.80	11.46
Citizens	7.03	8.24	Blue Island	6.25	11.63
University District	6.60	8.32	Atwood	5.70	11.66
Fidelity	6.55	8.34	Carrier Mills	10.41	15.14

Let x represent the investor's rate and y the borrower's rate in per cents. If one calculates the correlation coefficient it is found to be

[1] *Cf.* an article by the author in the *Bulletin of the American Mathematical Society*, Vol. XXXIII, No. 2, p. 209.

$r = .491 \pm .096$, which shows a poor correlation. If, however, one omits the five highest loan rates as unusual, one obtains $r = .775 \pm .056$, and the straight line of best fit by the method of least squares is $y - 8.15 = .69\ (x - 6.88)$. It is believed that these 28 companies represent a well distributed sample and that a study of their rates has a real significance for building and loan rates in Illinois. The inner meaning of the data may be exhibited in a two-way frequency table thus:

x	y										
	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	Total f_y
4– 5						(1)					0 (1)
5– 6	1	4				(1)					5 (6)
6– 7	3	2	4	1	(1)	(1)					10 (12)
7– 8			3		1						4
8– 9			1		1						2
9–10				1							1
10–11					1					(1)	1 (2)
Total f_x	4	6	8	2	3 (4)	(3)				(1)	23 (28)

The items excluded together with their totals are here shown in parentheses. They are omitted from consideration in the subsequent tables.

The data is consolidated further in a table with nine cells:

	y below 8	8–9	Over 9
x below 6	5	0	0
6–7	5	4	1
Over 7	0	4	4

The relation between the rates may also be studied by the method of averages by which the two-way table is summarized as follows:

x	Average y	y	Average x
5– 6	7.09	6– 7	6.06
6– 7	7.75	7– 8	5.87
7– 8	8.92	8– 9	7.00
8– 9	9.75	9–10	8.06
9–10	9.23	10–11	8.85
10–11	10.14		

These series may be used to test roughly the goodness of fit of the regression lines.

FACTORS PRODUCING UNEQUAL WEALTH DISTRIBUTION AND OWNERSHIP AND MEASUREMENTS OF THEIR INFLUENCE

By Le Grand Powers

The results of the late War have confirmed on a world-wide scale the truth of certain principles which lie at the base of the current teachings of economics. One of these principles is that economy in consumption, or savings, is an essential condition of the creation or accumulation of capital. By this economy, individuals acquire economic goods and, at the same time, wealth becomes divided or distributed among those saving. In this distribution the influence of many factors work together in creating and maintaining unequal wealth ownership among the masses. This inequality is everywhere a subject of universal interest. It has also been and is the cause or occasion of profound alarm on the part of many individuals. So apprehensive have some of these become that they behold in this inequality an indictment of our social institutions and a ground of grave fears concerning the future progress and welfare of mankind.

This paper is primarily a study of the factors referred to. It reviews the possible and probable influence of saving, under the operation of those factors or under certain conditions, upon the distribution of wealth among the people. It employs the words saving and savings with their broadest possible meanings and not with the narrow significance which they are given in some technical books on economics. As here used, an individual is said to be saving when his expenses for one or five years are less than his income and, as a result, he is worth more at the end of the given time than at its beginning. In like manner, the savings of the people of a nation for a decade or a century are the additions to their aggregate wealth during the stated period.

The part which savings have played and must always play in the creation of capital is treated at length in most textbooks on economics. As a rule, however, the student of the subject looks in vain in those books for any definite and thoughtful statement of the influence of saving by itself, or in connection with other factors, upon the distribution of wealth. The situation should be otherwise, since, without trustworthy information relating to those factors, no one can determine the justice or injustice, or even the actual significance of the existing allotment of wealth to individuals and families. And what is

of vastly greater importance, without this specified information, it is utterly impossible, intelligently and efficiently to legislate or work for the elimination or correction of the evils and wrongs in the prevailing wealth distribution. Further, without the data referred to, all legislation and work for removing unjust and mischievous inequalities must be carried on in the dark, with more than an even chance of producing evil rather than good and thus of multiplying rather than of eliminating injustice and wrong.

In this paper, use is made of certain curves or graphs to picture to the eye the facts presented in statistical tables. The graphs utilized are modifications of the well-known device worked out by Dr. Max O. Lorenz, at present Chief Statistician of the Interstate Commerce Commission.

In the chart here presented, line A is drawn at an angle of 45 degrees to the two axes. It represents an actual or hypothetical ownership of wealth which is equal as between individuals, or between classes of the same size. It represents an economic status in which 10 or 50 per cent of the persons or classes owning wealth, possess 10 or 50 per cent of the wealth, no more and no less.

Line C, drawn at the point of the horizontal axis most remote from its origin, represents that hypothetical economic status in which one individual owns 100 per cent of all wealth. With such ownership, all individuals save the one are without property or wealth. The line pictures graphically a social status towards which some individuals, alarmed at existing unequal wealth distribution, fear that modern society is inevitably drifting. It is a status the very antithesis of that represented by line A with its equality of ownership. With the specified antithesis, the space between lines A and C represents graphically and relatively all inequality of wealth ownership or distribution which is possible. That space may therefore be employed in measuring approximately the inequality of wealth distribution represented by any line drawn between lines A and C.

Line B, and all the curved lines here plotted, represent ownership and also distribution of savings or wealth among many. Accordingly they are most fittingly spoken of as "Lines of wealth distribution among many," or "Lines of wealth ownership by many." As all of the curved lines represent unequal ownership or distribution of wealth, they can be properly referred to as "Lines of unequal ownership or distribution of wealth."

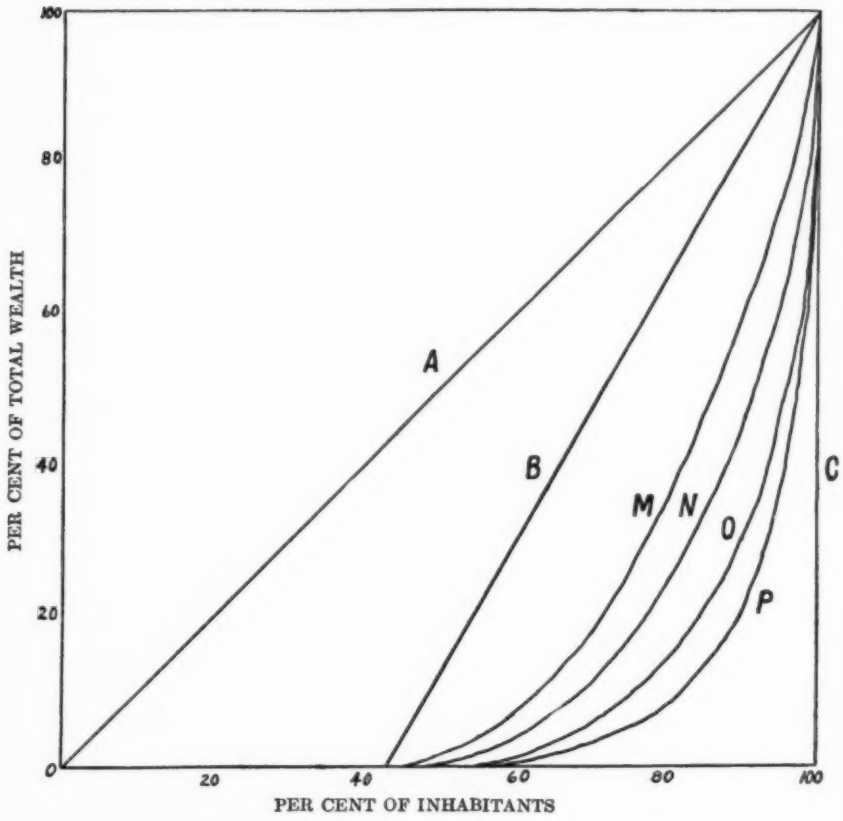
The degree of inequality of wealth ownership or distribution represented by any of these lines is indicated in the chart by departure of that line from line A towards line C. The relative magnitude of the

inequality of any case represented by a given line is measured by the space between the given line and line A as compared with the total space between lines A and C, the space which pictures relatively all possible inequality. The inequality thus measured may be expressed as a percentage of possible inequality.

Of the many legal provisions and social and political conditions affecting wealth ownership by individuals, none produce as much inequality in such ownership as does the existing organization of families and the laws relating to the property rights of the members of those families. The typical or representative family of the United States at this time is made up of one or two parents and two or three children. These children do not legally possess property until they attain their majority at their 21st birthday, all property being held for them until that time by their parents or guardians. In our several states, from 40 to 45 per cent of all individuals are under 21 years of age. The minors thus constitute a trifle more than 44 per cent of the population, and those 21 years of age and over make up nearly 56 per cent of all.

As a first hypothesis, for purposes of this study, let it now be assumed that all the wealth of the American people is owned or distributed equally among the 56 per cent who have attained their majority. To represent such ownership graphically, a straight line is drawn diagonally upward and to the right from point 44 of the horizontal axis (a point which depicts an ownership of nothing by 44 per cent of the individuals) to a point which pictures the ownership of 100 per cent of the wealth by 100 per cent of the individuals. This is line B. It is to be studied primarily in connection with line A, a line which represents an equal ownership of wealth among 100 per cent of our national population instead of a division of such ownership among 56 per cent as pictured by line B. The space between the two lines is 44 per cent of that between lines A and C, and hence the space first mentioned represents, according to the principles above set forth, 44 per cent of all possible inequality of wealth distribution. It thus graphically portrays the fact that substantially 44 per cent of all possible inequalities of wealth distribution among the individuals of the United States is caused by or results from the existing organization of families and by the ownership of all property by adults.

The words "Equal opportunity for the acquisition of wealth" have a meaning altogether different from that of the phrases "Equal wealth ownership," and "Equal wealth distribution." Individuals may have equal wealth without having had equal opportunities for its acquisition. In turn they may have had the same opportunities for acquisition without acquiring the same amount of riches. Notwithstanding this dif-



ference, the specified terms are often employed as if they were almost interchangeable. When joined with the terms wealth ownership and wealth distribution, the words, equal and equality, assume widely different meanings as has already been pointed out. When joined with the phrase equal opportunity for wealth acquisition, the same words, equal and equality, take on an even greater number of diverse meanings. Each of the hundreds of equal opportunities which are conceivable or which are assumed hypothetically for purposes of study of existing wealth ownership would be accompanied by a wealth ownership or distribution clearly differentiated from all others. The wealth ownership or wealth distribution thus resulting from various equal opportunities for wealth acquisition, if graphically pictured, would be represented in each case by a curved line. Some of the conceivable cases of such opportunities for wealth acquisition are very simple, while others are very complex. To reach the most trustworthy con-

clusion, a student of the subject must begin his investigations with the simplest possible case.

In the United States and in all other nations, the numbers of the people of two different ages are never the same. This statement is also true of typical or representative portions of the population. Under these circumstances, if all adult individuals were to save the same amounts each year, the annual accumulations of the several age classes would make not an arithmetic series but a progressive series such as is shown in Table I.

TABLE I

THE APPROXIMATE AGGREGATE AND AVERAGE WEALTH OF SPECIFIED PERCENTAGE CLASSES OF THE PERSONS 21 YEARS OF AGE AND OF THOSE OF ALL AGES IN A TYPICAL UNITED STATES POPULATION OF 1,000,000, TOGETHER WITH THE AGE LIMITS OF CLASSES AND PERCENTAGES OF THE TOTAL WEALTH WHICH IS OWNED BY EACH CLASS, *PROVIDED* EACH ADULT PERSON ANNUALLY ACCUMULATES $100

Per cents of persons			Age limits	Wealth of percentage classes				
				Classes not cumulative			Classes cumulative	
(a)	(b)	(c)		Aggregate	Average	Per cent of total	Aggregate	Per cent of total
		44.03	Under 21	0	0	0.00	0	0.00
10	10	49.63	21 to 23	$10,823,200	$193	0.97	$10,823,200	0.97
10	20	55.22	23 " 26	26,871,000	470	2.39	37,694,200	3.36
10	30	60.82	26 " 30	43,995,300	786	3.92	81,689,500	7.28
10	40	66.42	30 " 33	63,387,900	1,132	5.64	145,077,400	12.92
10	50	72.02	33 " 37	84,487,300	1,509	7.52	229,564,700	20.44
10	60	77.61	37 " 41	107,046,900	1,948	9.54	336,611,600	29.98
10	70	83.21	41 " 47	133,718,100	2,389	11.91	470,329,700	41.89
10	80	88.81	47 " 53	165,652,100	2,977	14.75	635,981,800	56.64
10	90	94.41	53 " 62	206,252,900	3,685	18.37	842,234,700	75.01
10	100	100.00	62 and over	280,629,500	5,014	24.99	1,122,864,200	100.00
5	95	97.21	62 to 68	126,561,700	4,528	11.27	968,796,400	86.28
5	100	100.00	68 and over	154,067,800	5,505	13.72	1,122,864,200	100.00
1	96	97.76	68 to 70	27,629,500	4,937	2.46	996,425,900	88.74
1	97	98.32	70 " 72	28,741,800	5,136	2.56	1,025,167,700	91.30
1	98	98.88	72 " 75	30,142,500	5,385	2.68	1,055,310,200	93.98
1	99	99.44	75 " 79	31,952,800	5,709	2.85	1,087,263,000	96.83
1	100	100.00	79 and over	35,602,200	6,360	3.17	1,122,864,200	100.00
0.5	99.5	99.72	79 to 83	17,018,600	6,105	1.51	1,104,281,600	99.34
0.5	100.0	100.00	83 and over	18,583,600	6,639	1.66	1,122,864,200	100.00

(a) Non-cumulative per cents of the 559,738 adults. (b) Cumulative per cents of the same. (c) Cumulative per cents of the 1,000,000 of all ages of whom 440,262, or 44.03 per cent, are less than 21 years of age.

Table I has been computed on the basis of the age distribution of the population of the United States in 1910,[1] or of a typical population of 1,000,000 at that time. In that population, there were 440,262 persons under 21 years of age and 559,738 who were of that age or older.

[1] The use of the figures for 1920 would not affect the results materially.

Further, the number of persons of any given age was somewhat less than for any younger age. The computations for the class wealth as presented in Table I are predicated upon the following assumptions concerning individual savings. Each person 21 years of age is assumed to possess $100. Each other person above that age is assumed to have the same amount multiplied by the number of years of his age above 20. The table gives the age limits of certain specified percentage classes and the percentages of the total wealth or savings belonging to the persons in each class. It also gives for all classes the aggregate and average wealth. It gives further in columns with the subheads (a), (b) and (c) the percentages of the population in the several classes. In columns (a) and (b) are non-cumulative and cumulative percentages of the 559,738 adults, while in column (c) are cumulative percentages of the 1,000,000 of which the adults form a part.

The cumulative percentages of the column of Table I with the subhead (c) and those of the same table relating to wealth ownership when plotted against one another result in line M of the chart. It graphically represents the unequal wealth distribution which would result from or follow the specified equal opportunity of wealth acquisition, namely an opportunity for equal annual individual savings by all those 21 years of age and over. Line M is here spoken of as "The curve of the influence of national age distribution upon wealth distribution among adults." It may also be called "The curve of the equal opportunity of equal annual savings by all adults." The percentages of Table I and the position of line M would be unchanged were the annual savings any amount other than $100.

The space between lines M and B constitutes 41.2 per cent of the space between lines B and C. Furthermore, the space between lines M and A constitutes approximately 64.1 per cent of the space between lines A and C. It thus is demonstrated that the inequality in wealth distribution among all individuals which would result from that equal annual opportunity for wealth acquisition represented by equal annual savings by all adults would be 64.1 per cent of all possible inequality among all our national population as well as 41.2 per cent of that possible among adults.

The inequality of wealth distribution considered in this paper is largely hypothetical. It is inequality conditioned upon certain provisions or assumptions. The importance of the figures presented and the validity of the conclusions deduced therefrom depend, therefore, in large measure, upon the actual relation of age and national age distribution to the wealth or economic status of the large masses of the American people. Does that wealth or economic status in the mass increase

or improve with the age of individuals, as is presupposed by the hypotheses here employed? If it does, the measurements of hypothetical inequalities here stated have a very important and vital relation to existing inequalities, otherwise not.

At present there are no wealth statistics giving for any large masses of the American people wealth classifications according to the age of the individuals owning the wealth. There are, however, available some rather old but, nevertheless, very illuminating statistics relating to the economic status of those millions of the inhabitants of the United States who gain their living and that of their families by working upon farms. These people constitute not far from one-fourth of the American population. The given statistics are contained in the report of the Twelfth Census, that for 1900. They are to be found in the subdivisions of the report relating to Proprietorship of Homes and to Occupations. Certain of those statistics are summarized and condensed in Table II. The census figures relating to homes with ownership not reported and to owners and workers with ages not reported are omitted. If adjustments were made for these omitted figures, the numbers of all classes and the percentages for farm owners would be slightly increased and the percentages for the others slightly decreased.

TABLE II

HEADS OF FARM FAMILIES AND OTHER ADULT EMPLOYEES ON THE FARMS OF THE UNITED STATES, JUNE 1, 1900, CLASSIFIED ACCORDING TO AGE PERIODS, WITH AGGREGATES AND PERCENTAGES FOR EACH AGE PERIOD

Age periods	Total number of farm workers	Heads of families with		Adult workers other than owners and tenants	Per cent of all falling in various age groups		
		Owned farms	Hired farms		Owners	Tenants	Other workers
All ages	7,833,771	3,631,993	2,005,063	2,196,715	46.3	25.6	28.1
21 to 24	1,071,068	76,452	198,696	795,920	7.1	18.5	74.3
25 " 34	2,055,296	541,237	653,505	860,554	26.3	31.8	41.9
35 " 44	1,709,214	908,514	501,689	299,011	53.2	29.3	17.5
45 " 54	1,434,111	916,607	379,839	137,665	63.9	26.4	9.7
55 " 64	933,117	683,635	181,345	68,137	73.3	19.4	7.3
65 and over	630,965	505,548	89,989	35,428	80.1	14.3	5.6

A glance at Table II suffices to show that, despite the presence of those incompetents and derelicts who are to be found on the farms, just as a submerged tenth is present in our cities, the economic status of nearly all of our farm workers was elevated with the passage from age 21 to age 70. To such an extent was this the case, that only 20 per cent of the farm workers reaching 70 years of age did not have some wealth in land. The farm owners were relatively twice as numerous among

those 70 years of age as among those from 21 to 24 years old. The wage earners who had reached three score and ten formed only one-thirteenth as large a proportion of all workers of like age as was true at age 24.

Here is a marked rise in the economic status of the great majority of farm workers with the passage of years. When the figures shown in Table II are considered in connection with other available facts relating to our farm population, it becomes certain that the great majority of farm owners, tenants, and wage earners increased their individual wealth even more than they changed their farm or economic status. As was assumed in the hypotheses for Table I, increasing years meant for all, with few exceptions, increasing wealth. Such a widespread elevation for one-third of our population indicates the probability of a like improvement for the great majority of the people in town as well as in country.[1] The hypothetical inequality of Table I and line M have, therefore, a very vital relation to existing unequal wealth distribution in the United States and in other countries.

Of the men and women of the nation who possess anything other than household goods, family clothing, and personal adornments, great numbers have the bulk of their wealth deposited with banks and building and loan associations. Others use their savings for the acquisition of farms and homes, or invest them in mortgages, bonds, and stocks of various kinds; while considerable numbers employ all their savings in the business enterprises in which they are engaged. In these, and in manifold other but associated ways, the great majority of adults secure some income in addition to that obtained as compensation for personal services. All such additional income is here referred to under the generic designation of interest and profits.

Now if every adult saved but $100 each year and succeeded in obtaining on his savings only 3½ per cent a year, a rate easily obtainable at savings banks, the degree of inequality would be that shown by Table III and line N.

Curves O and P represent the results of calculations similar to those recorded in Table III, but assume instead gains from accrued savings at the respective rates of 7 and 10 per cent instead of 3½ per cent.

The three lines of wealth distribution, N, O, and P, when compared with line M, demonstrate that the respective magnitudes of equal opportunities for wealth accumulation are factors producing unequal wealth distribution; the greater the opportunity, other factors being

[1] Investigations made in 1918 showed the continuance of this tendency for farmers to climb the economic ladder. See in this connection the paper by Richard T. Ely and Charles J. Galpin in *The American Economic Review Supplement*, March, 1919, pp. 180–211.

TABLE III

THE APPROXIMATE AGGREGATE AND AVERAGE WEALTH OF SPECIFIED PERCENTAGE CLASSES OF THE PERSONS 21 YEARS OF AGE AND OVER AND OF THOSE OF ALL AGES IN A TYPICAL UNITED STATES POPULATION OF 1,000,000, TOGETHER WITH THE PERCENTAGES OF THE TOTAL WEALTH WHICH WOULD BE OWNED BY THE SEVERAL CLASSES, *PROVIDED* EACH ADULT PERSON ANNUALLY ACCUMULATED $100 PLUS 3.5 PER CENT OF PRIOR ACCUMULATIONS

(NOTE.—The age limits of the several classes are the same as those given in Table I.)

Per cents of persons			Wealth of percentage classes				
			Classes not cumulative			Cumulative classes	
(a)	(b)	(c)	Aggregate	Average	Per cent of total	Aggregate	Per cent of total
		44.03	0	0	0.00	0	0.00
10	10	49.63	$11,065,156	$198	0.53	$11,065,156	0.53
10	20	55.22	28,204,750	504	1.35	39,269,906	1.88
10	30	60.82	50,388,746	900	2.44	89,658,652	4.32
10	40	66.42	76,397,805	1,365	3.67	166,156,457	8.99
10	50	72.02	108,927,686	1,946	5.24	274,984,143	13.23
10	60	77.61	139,960,870	2,500	6.74	414,945,013	19.97
10	70	83.21	213,397,147	3,812	10.27	628,342,160	30.24
10	80	88.81	282,687,119	5,050	13.60	911,029,279	43.84
10	90	94.41	410,275,246	7,330	19.75	1,321,304,525	63.59
10	100	100.00	756,564,313	13,516	36.41	2,077,868,838	100.00
5	95	97.21	305,699,501	10,923	14.71	1,627,004,026	78.30
5	100	100.00	450,864,812	16,110	21.70	2,077,868,838	100.00
1	96	97.76	71,488,660	12,772	3.44	1,698,492,686	81.74
1	97	98.32	77,666,176	14,052	3.74	1,776,158,862	85.48
1	98	98.88	85,934,649	15,347	4.14	1,862,093,511	89.62
1	99	99.44	97,240,352	17,372	4.68	1,959,333,863	94.30
1	100	100.00	118,534,975	21,177	5.70	2,077,868,838	100.00
0.5	99.5	99.44	56,808,720	20,298	2.73	2,016,142,583	97.03
0.5	100.	100.00	61,726,255	22,554	2.97	2,077,868,838	100.00

(a) Non-cumulative per cents of the 555,738 adults. (b) Cumulative per cents of the same. (c) Cumulative per cents of the 1,000,000 of all ages of whom 440,262, or 44.03 per cent, are less than 21 years of age.

the same, the greater the inequality. This demonstration is found in the greater curvature of the three lines N, O and P than that of line M and in the varying curvatures of the three specified lines increasing from line M to line P with the increase in the magnitude of the equal opportunity in the form of uniform annual savings.

Measured as in the case of line M, the inequality of wealth distribution resulting from the equal opportunity described in the title to Table III and pictured by line N is 73.5 per cent of the possible inequality for all, young and old, as compared with the inequality of line M of 64.1 per cent. The corresponding inequality indicated by line O is 81.0 per cent, and by line P is 86.7 per cent.

That American adults either save, on the average, less than $100 annually or receive on their savings less than the 7 per cent indicated by the hypothesis upon which curve O is based, is indicated by the fact that that rate of saving and that rate of increase on accumulated wealth

would give an average per capita wealth of $4,845. This is 2.48 times the national per capita of $1,950 in 1912, and 1.69 times the $2,918 representing the wealth per inhabitant in 1922 as estimated by the national Bureau of the Census.

As a matter of fact, with 7 per cent addition each year upon prior accumulations, a basic saving of $40 annually would have sufficed to give the national per capita of $1,950 existing in 1912. With 10 per cent on prior accumulations, the per capita wealth with a regular annual saving of $100 would be $12,224. A basic saving of only $16 per annum for all adults with the specified per cent of annual additions would have sufficed to produce an average wealth of $1,965, an amount greater than the average existing in 1912. These facts show something of the power of universal annual saving coupled with a high interest return on investments.

The $100 basic saving of each year assumed for Table III is an amount which the ordinary able-bodied, healthy, intelligent working man or woman, who is willing to live on the ancestral level, can save from the compensation which he or she receives for services. The percentage addition allowed in the given table with the specified basic saving is 3½ per cent per annum. This percentage is probably below the average rate which savings-banks depositors have been receiving for the past 15 or 20 years. It is also a rate which can be secured on the safest kinds of investments. The average wealth which would be accumulated by a basic saving of $100 annually by all adults with 3½ per cent annual additions on all prior accumulations would be $2,078, or a trifle more than the national per capita of 1912. The figures of Table III thus represent the outcome of an equal opportunity which was actually open for all able-bodied and intellectual persons of 21 years of age and over who were possessed of the spirit of thrift in excess of his desire to elevate and improve his or her standard of living.

The facts hitherto reviewed make it practically certain that, did all adult people possess and utilize the opportunities for saving represented by the curved line N and which are now open to the able-bodied and intelligent with thrift, the resulting inequality of wealth distribution would be about 73 per cent of possible inequality. But many people are living in our midst who are not able to save $100 per annum. Others, who are able, elect to use a part or the whole of their opportunity for wealth acquisition for purposes and objects which are personal to themselves. A third but smaller class enjoy and utilize opportunity for greater acquisition than is indicated by Table III.

The individuals who, by no act of their own, find it impossible to save $100 annually plus 3½ per cent of prior accumulations are readily

separable into three sub-classes as follows: (1) The utterly incapables, including the feeble-minded, the insane, and those totally blind, and others who are without the power, ability, or intelligence to save or even to earn much of anything; (2) those suffering from accidents, sickness, and other unavoidable calamities partially or entirely destroying the earning as well as the saving ability of themselves and their families; (3) those with fair ability who by lack of suitable opportunities or from limitations in their power of initiative are able to earn or save no considerable amount.

The existence of these three sub-classes in society without question adds to the inequalities of wealth distribution which would follow the equal opportunity outlined in the title to Table III. The amount of this added inequality cannot be as great as that due to the second principal cause mentioned to which attention is more particularly called in the paragraphs which follow. It may be nearly as great as the inequality due to the exceptional opportunity which a limited number enjoy and which enables them to acquire very large fortunes.

This inequality can be lessened as the numbers of each of the sub-classes are decreased. It can be decreased by lessening the economic dependence of the members of sub-class (1); by removing the handicap now borne by the members of sub-class (2). This can be done by providing adequate accident, sickness, and other social insurance. The same sort of insurance combined with a better popular education can lift the third sub-class to newer and better opportunities, and give them greater ability to utilize those falling to their lot and in this way lessen the vicious inequality of wealth distribution now caused by the existence in society of those who cannot at the present time save the small amounts called for by Table III.

Mention has been made of the fact that a part of the extreme inequalities of wealth ownership is due to the voluntary choice of a considerable portion of modern society. This portion is constituted of the men and women who are able to save $100 annually plus 3½ per cent of prior accumulations, but who elect to use their opportunity for wealth acquisition for other purposes. These men and women are readily separable into two sub-classes. The first of these sub-classes is composed of enterprising, intelligent, and ambitious persons who use possible savings for their own education, or that of their families, or for securing otherwise the elevation of their standard of living. The second of these sub-classes is made up of the improvident.

The first sub-class constitutes the most useful part of modern society. It is the salt which prevents the debasing and putrefying accumulation and use of wealth for its own sake. It is the inspiration of all intel-

lectual, moral, scientific, and social progress. Its most prominent members are the men and women who live and work for others, for scientific research, for social betterment, for reform and the elevation of the masses. This sub-class is composed of the men and women who have caught in some measure at least the spirit of an intellectual and spiritual awakening. The inequality caused by the free choice of the first of the two sub-classes should be fostered by legislation, as that due to the second should be held in check.

The third class of individuals in modern society whose possessions assist in creating wide disparity in individual fortunes is composed of a comparatively small number of the very rich. Some of this class have accumulated all their possessions in their lifetimes. Others have received them mainly, if not wholly, from their forebears. The greater number of persons in modern society when thinking or discussing the great existing inequalities of wealth distribution act and speak of this small class as the factor principally responsible for all modern inequalities. The facts here reviewed demonstrate that, of the present inequalities of wealth ownership, this class is answerable for only a small part. It is not in the least answerable for the 80 per cent of unequal wealth distribution pictured by line N. It is at best or worst responsible for only a fraction of the remaining 20 per cent of extreme inequalities. It shares, however, the responsibility for that 20 per cent with the class with little or no opportunities for wealth accumulation. It must also share that responsibility with the class which, having opportunity for saving, decides wisely or unwisely to spend possible savings for other purposes. The inequality of wealth distribution due to the accumulation and existence of enormous fortunes, like that due to the choice of many to spend rather than to save, may be either a curse or a blessing. If it is the outcome of a genius for constructive management which creates great wealth as truly as does the labor of employees, it is a blessing. The accumulation of Henry Ford is of this type, and the resulting inequality of wealth ownership does injustice to no one, but instead contributes greatly to a higher standard of living among the masses. It is quite otherwise with fortunes large or small which represent the fruits of extortion and dishonest practices. Wise laws should be enacted which should limit the number and at last eliminate the fortunes of the latter class, while providing added opportunities for the creation of the former. In a word, whether dealing with the great capitalist and employer or the simple toiler, the state should not seek to secure equal fortunes for all, but to guarantee equal opportunities for creative work and the wider exercise of native ability and individual resources.

NOTES

WEEKLY WHOLESALE INDEX

SIXTH REPORT

By Irving Fisher

This index began with January, 1923. It then employed weights (quantity coefficients) based on those of the War Industries Board. These were for the year 1917.[1]

In 1924, I revised the weights to conform with those of the United States Bureau of Labor Statistics based on the census data for 1919.[2] These same weights, with minor changes, were used throughout 1925, '26, and '27.

From the beginning, my index was a continuous one, spliced on to the monthly index of the United States Bureau of Labor Statistics. The splicing was first made as of November, 1922, when the index of the United States Bureau of Labor Statistics stood at 156. That is, the *base period* was the central week of the month, ending November 17, and the *base number* of the new weekly index was taken as 156.

On this base number the index a year later (for the week ending November 16, 1923) was 151.9. This, in turn, was taken as the base number for 1924, and the index for mid-December, 1924, was found to be 155.6. On this new base number for 1925, the index number for mid-December, 1925, was 159.5. With this as the base number for 1926, the index for the last week of December, 1926, was 146.1. In like manner, taking 146.1 as the base number for 1927, the index for the last week of December of that year was 145.1.

Thus the weekly index, beginning with mid-November, 1922, was computed on a series or chain of intermediate bases, each fixed for one year.

The complete chain index series is given in Table I, together with the figures of the United States Bureau of Labor Statistics for the nearest comparable month.

The close correspondence of the two indexes independently computed is remarkable but not surprising.

Tables II, III, IV, and V bring down to the end of 1927 the weekly,

[1] See "A Weekly Index Number of Wholesale Prices," this Journal, September, 1923.
[2] See "Revision of the Weekly Index Number," this Journal, September, 1924.

TABLE I

SELECTED WEEKLY INDEX NUMBERS
FISHER AND UNITED STATES BUREAU OF LABOR STATISTICS

	Fisher's Index (Weekly)	U. S. B. L. S. Index (Monthly)
Week ending November 17, 1922	156.0	156.0*
" " November 16, 1923	151.9	152.1
" " December 19, 1924	155.6	157.0
" " December 31, 1925	159.3	156.2
" " December 31, 1926	146.1	147.2
" " December 29, 1927	145.1	149.1

* This figure has since been revised by the United States Bureau of Labor Statistics to 155.5.

monthly, quarterly, and yearly index number as computed before the revision of weights and base year inaugurated at the beginning of 1928.

The group indexes are omitted as they would require too much space. They can be obtained from The Index Number Institute, New Haven, Connecticut, or from the files of any of the 100 newspapers which now take the service and in which the index appears Monday mornings.

TABLE II

WEEKLY INDEX NUMBERS
(Continuation of Table I, this JOURNAL, June, 1927, p. 222)

For week ending	Index number (1913=100)	For week ending	Index number (1913=100)
1927 (*continued*)			
May 13	139.6	September 9	144.4
May 20	140.0	September 16	144.6
May 27	139.9	September 23	144.7
June 3	140.1	September 30	145.5
June 10	140.4	October 7	145.5
June 17	140.4	October 14	146.2
June 24	139.3	October 21	146.2
July 1	138.8	October 28	145.0
July 8	139.9	November 4	145.2
July 15	140.5	November 11	145.7
July 22	137.9	November 18	145.9
July 29	139.5	November 25	146.1
August 5	138.7	December 2	145.7
August 12	138.6	December 9	146.1
August 19	139.9	December 16	144.3
August 26	142.0	December 23	145.1
September 2	142.1	December 30	145.1

TABLE III

MONTHLY AVERAGES
(Continuation of Table II, this JOURNAL, June, 1927, p. 222)

	Index number (1913=100)
1927 (*continued*)	
April	140.0
May	139.9
June	139.8
July	139.4
August	141.1
September	144.7
October	145.6
November	145.8
December	145.2

TABLE IV

QUARTERLY AVERAGES

(Continuation of Table III, this JOURNAL, June, 1927, p. 223)

	Index number (1913=100)
1927 (*continued*)	
1st Quarter	142.9
2nd Quarter	139.9
3rd Quarter	141.7
4th Quarter	142.6

TABLE V

YEARLY AVERAGES

	Index number (1913=100)
1923	157.7
1924	149.3
1925	159.2
1926	151.3
1927	141.7

At the beginning of 1928 I transferred my index number to the base year 1926 = 100 with weights of 1923, 1924, and 1925 to conform to the United States Bureau of Labor Statistics index.[1]

The weights adopted were adjusted by factors or "class weights" in order to give due importance to those groups for which there would otherwise be inadequate representation. The final weights thus obtained were rounded off to save time in calculation, without sacrifice of accuracy.

The quotations used, except bread, milk, and cotton, are, as before, taken from *Dun's Review*. About 80 items previously included are now omitted because their weights were practically zero and their influence on the index negligible. This leaves 120 series of price quotations which represent with surprising accuracy the 550 price quotations used by the United States Bureau of Labor Statistics for computing that Bureau's index numbers.

The articles are now classified into groups and fourteen separate sub-indexes and group indexes are calculated, namely: Agricultural Products and Non-agricultural Products; Raw Materials, Semi-Manufactured Goods, and Finished Goods; Farm Products, Foods, Hides and Leather Products, Textile Products, Fuel and Lighting, Metals and Metal Products, Building Materials, Chemicals and Drugs, and Miscellaneous.

[1] See "Revised Index Numbers of Wholesale Prices 1923 to July, 1927," United States Bureau of Labor Statistics, Bulletin No. 453, September, 1927.

Following the practice of the United States Bureau of Labor Statistics, some articles appear in more than one group. Four articles appear in both the Farm group and the Food group, while three appear in both the Metal group and the Building Materials group. This practice of repeating articles in more than one group is statistically sound, provided the duplicated articles are counted but once in the total index, which practice is followed in the Bureau and in my own office.

The values of the commodities in each of the several groups, as calculated by the United States Bureau of Labor Statistics, are shown in Table VI. It will be noted that the number of articles in the several groups totals 127. This is because of the repetition of seven articles in more than one group.

TABLE VI

	$(—000,000)
Farm products (15 articles)	9,412
Foods (19 articles)	12,652
Hides, leathers (8 articles)	1,622
Textile products (16 articles)	3,710
Fuel and lighting (10 articles)	7,153
Metals and metal products (18 articles)	5,834
Building materials (20 articles)	2,837
Chemicals and drugs (14 articles)	860
Miscellaneous (7 articles)	2,824

This new index, computed on the base 1926 = 100 with weights of 1923–1925, was calculated for the first three and one-half months of 1928 by the old method. That is, the index number for the last week of December, 1927, was used as the new "fixed-intermediate" base for computing the index-links week by week, and the index-links so computed were linked to the fixed base (1926 = 100) by multiplying the links into the index number for the last week of December, 1927.

It was my intention to make my weekly indexes a precursor or anticipator of the index of the Bureau of Labor Statistics. However, from the beginning my monthly aggregative indexes differed appreciably from those of the Bureau, and these differences showed a tendency to increase. A new method of computation was, therefore, introduced in April, by which my indexes are connected more directly with the Bureau's. Instead of using the last week of the year as a "fixed-intermediate" base of calculation, it was decided to use a "quarterly moving-base" as the intermediate step in calculating the weekly indexes. In brief, the method is to calculate an index-link for each week by dividing the aggregate for any given week by the average of the weekly aggregates for the latest three-month period for which the Bureau's indexes are available. This index-link is multiplied into the index number computed from the Bureau's monthly indexes for the

latest available three months in order to shift it to the base 1926 = 100. Thus my weekly indexes are brought into harmony with the Bureau's monthly indexes. Each month a new three-months moving base is substituted for the previous one, so that the latest published index of the Bureau is utilized to adjust my indexes more closely to the Bureau's.

This monthly adjustment of my weekly indexes with the monthly indexes of the United States Bureau of Labor Statistics may be roughly likened to a passenger on board ship adjusting his watch to the ship's chronometer. The monthly adjustments are very small. In ten weeks no adjustment factor was required. These monthly adjustments make certain that my weekly indexes can never wander very far from the Bureau's monthly indexes.

The new series is presented in Tables VII, VIII, and IX.

TABLE VII

WEEKLY INDEX NUMBERS, 1928

For week ending	Index number (1926=100)	For week ending	Index number (1926=100)
January 6	95.5	May 25	98.8
January 13	95.1	June 1	98.0
January 20	95.7	June 8	97.6
January 27	95.5	June 15	97.8
February 3	96.0	June 22	97.9
February 10	97.0	June 29	98.4
February 17	96.8	July 6	98.8
February 24	96.6	July 13	99.3
March 2	96.9	July 20	99.9
March 9	96.8	July 27	99.9
March 16	97.3	August 3	99.7
March 23	98.2	August 10	99.4
March 30	98.1	August 17	100.1
April 6	97.6	August 24	99.9
April 13	98.9	August 31	100.0
April 20	99.4	September 7	99.7
April 27	99.7	September 14	100.3
May 4	99.8	September 21	99.8
May 11	99.6	September 28	99.3
May 18	99.6		

TABLE VIII

MONTHLY AVERAGES, 1928

	Index number (1926=100)	U. S. B. L. S. Index number
January	95.5	96.3
February	96.7	96.4
March	97.6	96.0
April	99.1	97.4
May	99.1	98.6
June	98.0	97.6
July	99.6	98.3
August	99.8	98.9
September	99.7	100.1
October	98.8	....
November	....	....
December	....	....

TABLE IX

QUARTERLY AVERAGES, 1928

	Index number (1926=100)
1st Quarter	96.6
2nd Quarter	98.7
3rd Quarter	99.7
4th Quarter	....

The figures of Tables VII, VIII, and IX can be converted to the 1913=100 base by multiplying by 1.513.

As stated above, the weekly index is intended to be a sort of precursor, one month ahead, of the index of the United States Bureau of Labor Statistics. The close correspondence of the two indexes is shown in Table VIII.

THE RELATIONSHIP IN A FREQUENCY POLYGON OF THE AVERAGE OF ANY CLASS TO THE MID-POINT OF THE CORRESPONDING CLASS-INTERVAL

By Willford I. King, *New York University*, and Lucile Lawrence, *Columbia University*

In preparing for the National Bureau of Economic Research estimates of the distribution of earnings and income in the United States, it was found that there was a recurrent need for a formula showing the relationship between the mid-point of any given class-interval in a frequency table and the average size of the items in that class. While this relationship may have been worked out many times, the formula for the same does not appear in the texts or handbooks examined. As the solution is comparatively simple and the results are of decided practical value to those working on problems of distribution, it seems worth while to present the formula and explain its derivation.

By using this formula, it is possible in calculating, for example, the total income in any income class to secure results having margins of error far smaller than those obtained by the customary method of multiplying the mid-points of the various class-intervals by the corresponding class-frequencies. Furthermore, the formula has the great merit of being so simple that it can be applied with a minimum of effort.

The multiplication of the mid-point by the class-frequency involves the assumption that all the incomes in the class are clustered at the mid-point of the class-interval. An examination of actual data shows that, in practice, this assumption is usually contrary to fact. Generally, when the frequency is increasing, more items are above than below the mid-point—and vice versa. As a result, in the rising parts of a frequency curve, the class-averages tend to be higher than the class-mid-points, while, on the downward slope of the curve, the opposite relationship holds.

The errors on one slope tend to offset each other perfectly *when all class-intervals are equal*, so that, under such circumstances, the correct aggregate for the entire distribution may be obtained simply by multiplying the class-frequencies by their respective mid-points, and summating the products thus obtained. However, the correctness of this aggregate should not be allowed to hide the fact that all, or at least most, of the calculated class-totals are in error, those below the mode tending to be too small and those above too large.

In order to find the true relationship between the class-average and

the class-mid-point, let us assume the most probable arrangement of the items within the class—namely that the individuals are so spaced throughout the class that, if the class is divided into s sub-classes, each having a class-interval of h, the frequencies of these sub-classes will increase by equal increments, d, throughout the entire original class-interval. This would be the distribution represented by a frequency polygon made up of straight lines connecting the class-limits. For illustration we may imagine a class containing 77 persons between the limits of \$100.50 and \$121.50. Readings from a curve give relative numbers indicating the following frequency distribution. We will, for the moment, assume the incomes to be of exactly the sizes specified.

Income	Number of persons	Total income of class
\$102	8	\$816
105	9	945
108	10	1,080
111	11	1,221
114	12	1,368
117	13	1,521
120	14	1,680
	77	8,631

The average income of persons in class is $\frac{\$8631}{77} = \$112.09$.

We may now proceed algebraically to find the average income of all the persons in this class.

s = number of sub-classes (in illustration $s = 7$)

$n = \frac{s-1}{2}$ = number of sub-classes on each side of middle sub-class (in illustration $n = 3$)

$s = 2n+1$ (follows from the preceding)

c = class-interval of the original class (in illustration $c = 21$)

$h = \frac{c}{s}$ = class-interval of each sub-class (in illustration $h = 3$)

i = increase in frequency, as indicated by a smooth curve, between limits of original class (in illustration $i = 14\frac{1}{2} - 7\frac{1}{2} = 7$)

$d = \frac{i}{s}$ = difference in frequency between adjacent sub-classes (in illustration $d = 1$)

m = mid-point of original class-interval (in illustration $m = 111$)

f=frequency of middle sub-class (in illustration

$$f=\frac{\text{class frequency}}{s}=\frac{77}{7}=11)$$

a=average income of all persons in original class

Then

$$a=\frac{fm+(f-d)(m-h)+(f-2d)(m-2h)\ \ldots\ +(f-nd)(m-nh)}{+(f+d)(m+h)+(f+2d)(m+2h)\ \ldots\ +(f+nd)(m+nh)}.$$

$$fs$$

By multiplication and collection of terms this is reduced to

$$a=\frac{fm+2n\,fm+2dh(1+4+9+16\ \ldots\ +n^2)}{fs}.$$

But Dr. Frank A. Ross has shown in this JOURNAL for March, 1925, page 77, that

$$\Sigma(1^2+2^2+3^2+4^2\ \ldots\ .\ +n^2)=\frac{2n^3+3n^2+n}{6}.$$

By substituting for the last term of the numerator and also for d and h one obtains

$$a=\frac{m(2n+1)}{s}+\frac{2ci}{s^2}\cdot\frac{2n^3+3n^2+n}{6fs}.$$

But $s=2n+1$.

Therefore, substituting and factoring gives

$$a=\frac{m(2n+1)}{2n+1}+\frac{ci(n^2+n)(2n+1)}{3fs^3}$$

$$a=m+\frac{ci(4n^2+4n)}{12fs^2}$$

$$a=m\frac{+ci[(2n+1)^2-1]}{12fs^2}$$

$$a=m+\frac{cis^2}{12fs^2}-\frac{ci}{12fs^2}$$

$$a=m+\frac{ci}{12f}-\frac{ci}{12fs^2}.$$

If we substitute the quantities used in the illustrative frequency table we get the following results:

$$a=111+\frac{21\times7}{12\times11}-\frac{21\times7}{12\times11\times49}.$$

$a=111+1.11-.02=112.09$, the average income in this class.

But this is the identical result obtained by summating the sub-class products and dividing the aggregate by the total frequency of the original class.

Evidently the last of the three terms in the formula is negligible in size and of no practical significance. Moreover, its existence is due solely to the fact that all items have been assumed to lie at the mid-points of the sub-classes. If, on the other hand, we adopt the more probable assumption that the items are distributed throughout the sub-classes and hence are better represented by the averages than by the mid-points of the sub-classes, in other words, if s, the number of sub-classes, is imagined as increased indefinitely, then the third term approaches zero in size. Under such circumstances, the formula is simplified and we find that

$$a=m+\frac{ci}{12f}.$$

Substituting the quantities in the illustrative example, we have $a=111+1.11=112.11$, a quantity slightly more in accordance with the probabilities than the 112.09 previously obtained on the hypothesis that all items lay at the mid-points of the sub-classes.

If we multiply the equation by f we find

$$fa=fm+\frac{ci}{12}.$$

We now see why it is that, in a histogram representing a distribution with uniform class intervals,

$$\Sigma(fa)=\Sigma(fm).$$

Evidently, in this case, $\frac{c}{12}$ is a constant, and, since the sum of the positive i's must equal the sum of the negative i's, it follows that, under such circumstances, $\Sigma\frac{ci}{12}=0$.

In the practical application of the formula $a=m+\frac{ci}{12f}$, the readings for f and i are taken from a smooth histogram, and it is necessary to use at each mid-point the class-frequency as plotted rather than the frequency of a sub-class. However, the increase in the size of f is accompanied by an exactly proportional increase in the size of i, for the same scale used in reading the frequency at the mid-point of the class is also used in reading the frequencies at the class-limits, and i represents the difference between the frequencies at the class-limits. The

formula as stated above may, therefore, be applied unchanged to a frequency curve plotted on any scale.

To obtain the total for any class, it is, of course, only necessary to multiply the class-frequency by the class-average. If we consider, for example, the original classification given in the illustration

F = frequency = 77
a = average size of item = 112.11
T = total income in class = Fa = 8632.47.

ECONOMIC AND SOCIAL SURVEY OF LONDON

Most students of social questions are acquainted with the exhaustive survey of the economic and social conditions in London carried out by the late Mr. Charles Booth, and issued in a series of volumes from 1889 to 1902, under the title *Life and Labour of the People*, a pioneer work which has had a great influence on similar investigations during the past thirty years.

A committee has been organized at the London School of Economics to make a new survey of London that shall, on the one hand, be as closely comparable as possible with Mr. Booth's, and, on the other, by the use of information of a kind not available at the earlier date, to provide a new and firmer basis for subsequent investigations. The committee includes Sir W. H. Beveridge, Mr. Sidney Webb, and other prominent workers in the fields of sociology and statistics. The actual enquiry is directed by Sir H. Smith, recently retired from the Board of Trade, who was associated with Mr. Booth at the earlier date, assisted by Professor A. L. Bowley. It is intended to follow not only Mr. Booth's lines, and to make among other things a new map of London colored (as in the original) to show the degree and localization of poverty, but also to make intensive sample investigations on the method of *Has Poverty Diminished* (Bowley and Hogg), so that comparisons can be made with the towns treated in that book. It is also expected that investigations on similar lines will be organized in other towns.

The actual field work has recently commenced, and part of the new building at the School of Economics is now busy with organization, interviews and digestion of statistics.

A. L. Bowley

FREIGHT TRAFFIC OF CALIFORNIA CITIES

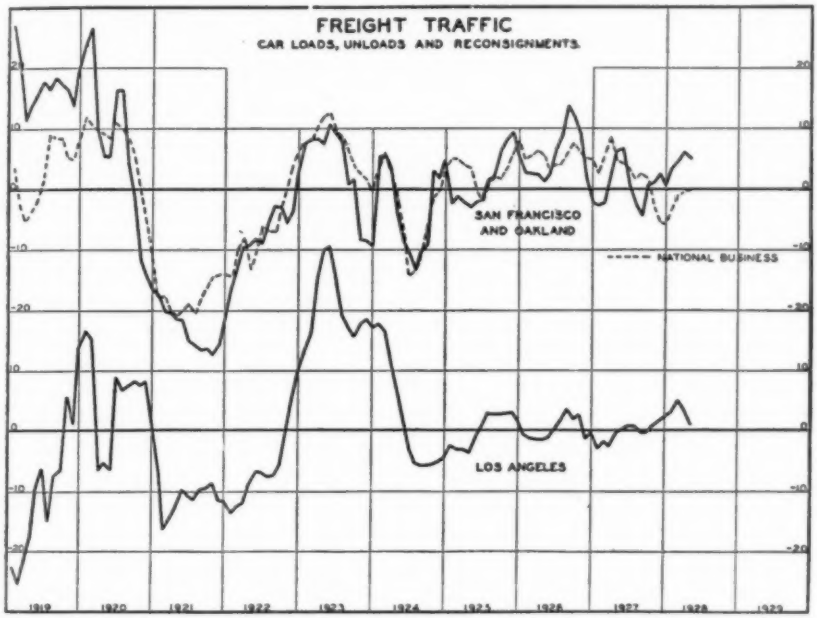
Car loadings have long been recognized as representative of general business conditions in a region. However, total freight traffic including car loads, unloads, and reconsignments has not been used as a measure of business for a limited metropolitan area. This series is now probably the best single indicator of business since speculative activity has so strongly influenced bank debits as to make them unreliable.

Total railroad traffic represents a composite of volume of production, distribution, and consumption, and gives weight to the distributive and consuming elements in the local business situation which are not adequately portrayed by car loadings alone.

The chart presented is based upon railroad freight traffic of San Francisco-Oakland and immediate vicinity, and of Los Angeles and suburban area. The seasonal fluctuations were computed from data for eighteen years. The trend for San Francisco-Oakland was a straight line fitted to the data from 1919–1927. Los Angeles trend was a five-year moving average projected as a straight line, since the actual figures for the last three years are identical to the nearest thousand.

The seasonal fluctuations for San Francisco-Oakland are rather extreme with a

difference of 41.4 points between the highest and lowest months. January and February are the months of least freight movement and August, September and October are the months of greatest activity, coincident with the greatest crop movements. The maximum variation in Los Angeles is 19.4 points. The



lowest months are April, May and June. October and November are the highest.

Conditions in the San Francisco Bay cities have followed national conditions closely as may be seen from the chart. The correspondence is also close between Los Angeles traffic and national conditions. The magnitude of fluctuations between the local and national indexes is much the same.

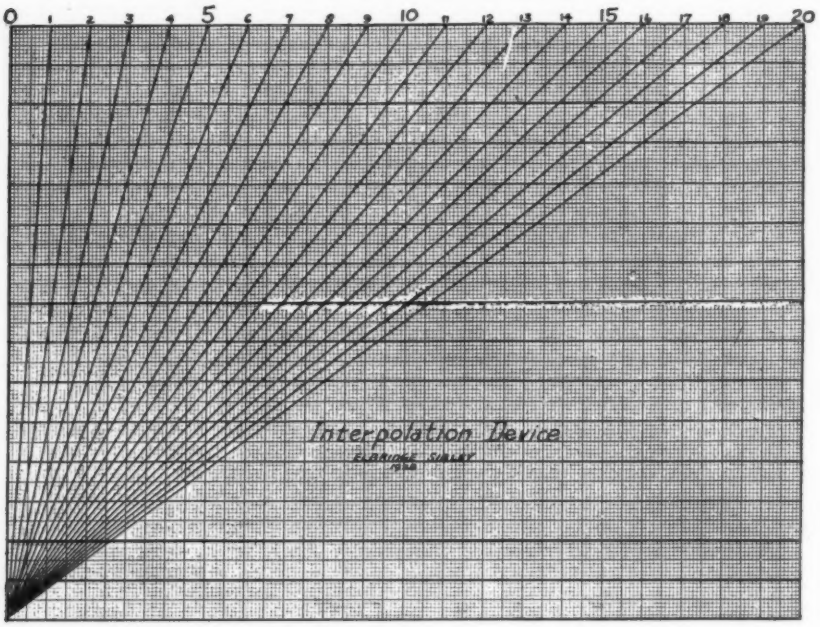
Situations peculiar to the smaller areas can be easily noted on the curve. The boom conditions in Los Angeles during 1923 and early 1924 stand out with especial prominence. The national index is an arithmetic average of two well known national indexes which have very similar movements.

ROBERT W. BACHELOR

A GRAPHIC DEVICE FOR ARITHMETIC OR GEOMETRIC INTERPOLATION

The device illustrated herewith has been of service to the writer in various problems in which it was desired to interpolate annual estimates between decennial figures, where accuracy beyond two or three places was not necessary. It consists simply of a piece of cross section paper on which are laid off a number of

lines radiating from a point on one margin to a number of points equally spaced along the opposite margin. By laying a scale on the paper horizontally and moving it up and down until the necessary distance is found between the radial



lines, it is possible to divide any interval on the scale into any desired number of smaller intervals. Any decimally divided scale may be used for arithmetic interpolation. By using the slide from a slide-rule, or the edge of a piece of logarithmic or "ratio" paper, intermediate points in a geometric series may be found with equal ease.

ELBRIDGE SIBLEY

PROGRESS OF WORK IN THE CENSUS BUREAU

METROPOLITAN DISTRICTS

It has long been recognized that the growth and importance of cities in the United States cannot be adequately portrayed in the census by giving simply the population within their corporate limits. More and more population growth is coming to be a growth of suburbs, with the result that in almost every sense except the political one the city extends beyond its corporate boundaries. In many cases two or more cities grow up side by side or in close proximity until they practically coalesce; or we may have a group of cities forming a single metropolitan aggregate.

Realizing this situation the Bureau of the Census in 1910 and again in 1920

published totals representing the population of the corporate city and the adjacent territory for all cities having over 100,000 inhabitants. The rule followed was to include as "adjacent territory" all political units, such as cities, townships, precincts, or boroughs, located within ten miles of the city boundary. Then for all cities having over 200,000 inhabitants a "metropolitan district" was defined, which included all contiguous political units located within the adjacent territory, as already defined, and having as a rule a density of population of more than 150 per square mile.

In preparation for the coming census of 1930 a new procedure is being followed. Through the United States Chamber of Commerce, the local chamber in every city of over 50,000 inhabitants has been invited to furnish the Bureau of the Census with a map of the city and surrounding territory outlining various areas, such as the area of daily commuting, the area of retail store free deliveries, the area served by electric light and power from the central city, the area served by the water-supply system, and finally, the area, which, having regard for all the various factors that ought to be considered, constitutes in the opinion of the chamber of commerce the metropolitan district. Of course the Bureau has not agreed to accept the district as defined by the local chamber of commerce. Without perhaps applying any rigid rule, the various districts in the end will have to be established on some fairly uniform or consistent principles.

It may be well to add that it is not proposed to substitute the metropolitan district for the corporate city. The population of any given city will continue to be, as it has been heretofore, the population within its corporate limits. The Census Bureau cannot disregard the existing political units or create new ones. But the population of the metropolitan districts will be shown in the census reports as additional information of value for indicating the actual situation as regards the growth and magnitude of cities.

CENSUS MONOGRAPHS

The following census monographs have been published thus far:

I. Increase of Population in the United States, 1910–1920. By William S. Rossiter.
II. Mortgages on Homes in the United States, 1920. A report prepared in the Bureau of the Census.
III. The Integration of Industrial Operation. By Willard L. Thorp.
IV. Farm Tenancy in the United States, 1920. By E. A. Goldenweiser and Leon E. Truesdell.
V. School Attendance in the United States, 1920. By Frank Alexander Ross.
VI. Farm Population in the United States, 1920. By Leon E. Truesdell.
VII. Immigrants and Their Children, 1920. By Niles Carpenter.
VIII. Growth of Manufactures. By Edmund E. Day and Woodlief Thomas.

The following monographs are in preparation at the present time:

Earnings of Factory Workers. By Paul F. Brissenden. (In type)
Women in Gainful Occupations. By Joseph A. Hill. (In type)
Ratio of Children to Women. By Warren S. Thompson. (In manuscript)

Of the eleven authors of the monographs thus far published or now in preparation, only two are on the permanent staff of the Bureau; six are college professors or instructors; two, one of whom was formerly an employee of the Bureau, are connected with another Department of the Government; and one, formerly an official of the Bureau, is in business.

In order to compensate the writers of these monographs they have, in one or two instances, been put on the pay roll of the Bureau temporarily as expert special agents. But usually they have been compensated by the payment of a lump sum agreed upon under contract. In order to provide for this form of payment, it was found necessary to have a special provision in the appropriation act, authorizing the expenditure of a certain limited sum for "the employment by contract of personal services for the preparations of monographs on census subjects."

The amounts paid to the authors constitute, however, only a small part of the expense of preparing these monographs; for, in order to bring out the significance of the census data in relation to the subject of which the monograph treats, it is always necessary to compute numerous percentages and averages and sometimes to make extended special tabulations. The latter may be based upon statistical tables published in the regular reports, but not infrequently the basic data have to be taken from the original census schedules, or from punched cards, or from unpublished tabulation sheets existing as a by-product of the tabulations for the main census reports.

Most of the monographs, it may be noted, relate to subjects covered by the population census. One of them, however, is based on the agricultural census, two upon the biennial census of manufactures, and one presents data obtained from a special questionnaire that was sent out by the Bureau in regard to mortgages on homes other than farm homes. It is obvious that within the wide range covered by the census statistics there are plenty of subjects still available for monographic treatment. No monographs have been produced in the field of vital statistics or in that of financial statistics of states and cities. The limitations upon the continuation or extension of this feature of the Bureau's work certainly do not result from any lack of subjects nor perhaps from any dearth of qualified writers; but they result rather from the limitation of the clerical and supervisory force that can be used for making the necessary tabulations required in the preparation of monographs without interference with the regular periodical work of the Bureau. It is probable that for the next two or three years and perhaps until we have completed the 1930 census, the work on monographs will have to be narrowly limited, if not completely suspended.

J. A. H.

MISCELLANEOUS NOTES

Program of the Annual Meeting of the Association.—The preliminary arrangements for the program of the Annual Meeting, at the Hotel Stevens in Chicago, include the following topics:

(1) Employment data and their meaning, and proposals for their improvement, with discussions of new indexes.
(2) The new Institutes of Industry and their progress, looking towards the statistical control of industry.
(3) The new distribution of gold and its probable effects.
(4) Statistical adjuncts in business management; the utilization of statistics in business planning.
(5) Regional indexes of the Volume of Trade and measures of national trade.
(6) Statistics of prohibition and crime; their value and meaning: a debate.
(7) Stock speculation and the expansion of credit.
(8) The stabilization of farm prices and the McNary-Haugen Bill.
(9) Banking policy and the business cycle.
(10) Recent experiments in index number making, their defects and advantages.
(11) Multiple correlation in business and crop forecasting.
(12) The measurement of the value of money.

There will be the usual joint sessions with the various other organizations meeting at Chicago, and a feature will be a lecture by Dr. Harlow Shapley, Director of the Harvard Astronomical Observatory, on "The Influence of Statistical Methods on Recent Scientific Development, With Especial Reference to Recent Progress in Astronomy," with lantern slides.

The meeting will open on Thursday morning at ten o'clock, December 27, and close with the Saturday afternoon session. A special business men's luncheon will be given on Saturday, to which the public is invited—Topic: "Prospects for Prosperity." Wednesday evening, officers and ex-officers, and district secretaries will hold a "get-together" dinner, for a general discussion of the affairs of the Association.

Following the policy inaugurated last year the papers will be printed as fully as possible in a volume of *Proceedings* to be issued as a supplement to the March, 1929, issue.

United States Bureau of Labor Statistics.—In the September issue of the *Monthly Labor Review* appears a study of time and labor cost of production in the woolen and worsted industry of the United States, England, France and Germany. This publication will later be available as a separate pamphlet. The work was done by Assistant Commissioner Charles E. Baldwin, who spent four months in Europe gathering the data. The plan was to take samples of American goods, having first ascertained all the details of time and labor cost for these samples both as to yarn and to cloth, and then compare this information with like information for pieces of cloth as nearly identical with the American samples as could be found in the European countries visited.

The Bureau of Labor Statistics is either engaged in or has finished the field work on investigations of wages, hours, and conditions of labor for 1928 in the following industries: Boots and shoes, men's clothing, lumber, automobiles, hosiery and underwear. Field work for the first three studies listed has been completed, while the other two investigations are now under way.

An investigation of the cost of living among Government employees outside of

Washington, D. C., is being undertaken by the Bureau of Labor Statistics at the request of the Personnel Classification Board.

A pamphlet has been issued on "Revised Index Numbers of Wholesale Prices from January, 1913, to June, 1928," constructed on the 1926 base period. In other words, it converts so far as is physically possible the index numbers of wholesale prices published on the old 1913 base to the 1926 base.

It will be interesting to know that at the fifteenth annual convention of the International Association of Industrial Accident Boards and Commissions held at Paterson, New Jersey, September 11–14, 1928, an entire session was devoted to the subject of occupational diseases growing out of the change in methods of industry. Dr. G. H. Gehrmann of the E. I. Du Pont de Nemours and Company in a paper showed "How Chemistry Has Changed Industry and as a Result Has Changed the Tendency to Occupational Disease." This was followed up by a paper by Mr. J. J. Bloomfield of the United States Public Health Service, while Dr. E. B. Patton of the New York Department of Labor described the new problems which were coming up before compensation commissions as a result of changes in processes of manufacture. A comparatively few states compensate for occupational diseases, and most of those specify just what diseases shall be compensated for. The new industry will make necessary an entire revision of workmen's compensation. A girl who once would have gotten a blow from a flying object to break her arm now gets a shock from radium which impairs her health for life. The subjects covered were exceedingly interesting and the discussion brought out an astonishing number of cases where chemical action had taken the place of mechanical action, revolutionizing the whole compensation theory.

Employment Certificates.—The Children's Bureau has completed the assembling of reports on employment certificates issued to working children during the year 1927, which will make available current information in regard to the trend of child labor and the age and education of children going to work. This assembling of reports is an annual activity of the Bureau. Reports for the year 1927 were received from the District of Columbia, 16 states, 69 cities in 18 other states, whereas for 1926 reports were received from the District of Columbia, 12 states and 26 cities in 11 other states. An estimate based on the 1920 Census indicates that more than half the 14- and 15-year-old children at work in occupations for which permits are usually required are employed in cities and states from which the Bureau received reports for 1927.

The reports have been tabulated for all states and for cities with a population of 50,000 or over to show the number of certificates issued in each place, the ages and school grades completed by the boys and girls receiving certificates, the occupations entered and the evidence of age accepted by issuing offices.

A general summary of the facts for 1927, as compared with those of 1926, appears in the annual report of the Chief of the Children's Bureau. More detailed analysis will be published by the Bureau at an early date.

United States Bureau of Agricultural Economics.—Among the important studies that have recently been made by this Bureau is the publication "Index Numbers of Prices Farmers Pay for Commodities Purchased," by C. M. Purves. This gives a complete presentation of the index together with the methods used in its preparation.

The last Congress added $70,000 to the appropriation for the Department of Agriculture for researches in agricultural transportation and prices, and for agricultural outlook work. The Bureau of Agricultural Economics is strengthening its staff for this work, and is employing Thor Hultgren, Instructor in Economics, Rutgers College,

New Jersey, for research in transportation, and Warren C. Waite and Elmer J. Working, Associate Professors in Agricultural Economics, University of Minnesota, for price analysis and outlook work.

An interesting feature of the work to be undertaken by Dr. Waite is an analysis of the demand for agricultural products. He will collect data as to consumption and retail prices of farm products and study the relation between prices and consumption in an effort to obtain fairly definite measures of the demand for specific farm products and for groups of farm products.

Developments in Canada.—The Dominion Bureau of Statistics, Ottawa, has recently been removed to a new and up-to-date building, secured and made over by the Government so as to permit of modern and efficient control of staff operations.

Canada during the last quarter joined the list of nations which attempt to draw up a Balance of International Payments and which attempt to estimate capital movements. According to the Dominion Bureau of Statistics' estimate, Canada had a net export of capital to the amount of $177,000,000 in 1925 and $85,000,000 in 1926. Total foreign investment in Canada is estimated at $5,500,441,000, which represents 33⅓ per cent of the total capital invested in Canadian industries, or 21½ per cent of Canada's estimated national wealth ($25,673,174,000).

There is about to be issued by the Bureau the results of a census of Trading Establishments in Canada taken in 1924, which covers wholesale and retail establishments all over the Dominion. Sixty-seven thousand stores, from which returns were received, had a capital investment of $1,580,000,000 and sales amounting to $2,300,000,000.

An index number of wholesale prices covering the period 1867 to 1890 has been calculated for the Dominion Bureau of Statistics by Mr. Herbert Marshall. The Bureau's index numbers are now available from Confederation to date.

Industrial Accident Statistics in Pennsylvania.—Plans are nearing completion to extend the statistical work of the Bureau of Statistics of the Department of Labor and Industry of the State of Pennsylvania so that, beginning January, 1929, it will be able to furnish the Inspection Bureau of this department each month with records which will show the number of accidents that have occurred in each industrial establishment of the state during the month immediately preceding. This is an important development in the statistical work of the department, and appears to be a new venture among state labor departments. The development of accident data of this kind is the result of persistent efforts on the part of Mr. Harry D. Immel, Director of the Bureau of Inspection, to secure accident records which would definitely indicate the individual establishments that are having bad accident records. By providing the Inspection Bureau with current information concerning the accident experience for individual industrial establishments, the work of the inspectors can be based primarily on the accident experience in individual establishments, and the unsatisfactory plan of block or sectional inspection work can be abandoned. This new method undoubtedly will be productive of good results in accident prevention work because inspection activities can then be concentrated on the plants showing the need for close supervision by the state inspectors. It is anticipated that the individual accident experience for some 50,000 industrial establishments in Pennsylvania will be currently available after this new work has been started.

Statistics of Welfare in Pennsylvania.—A Bureau of Social Statistics has been organized in the Public Charities Association of Pennsylvania, Social Service Building,

311 South Juniper Street, Philadelphia, for the collection and study of statistics of social work and welfare in Pennsylvania. The Bureau is under the part-time direction of Ralph G. Hurlin of the Russell Sage Foundation. Other members of the staff are Dr. Hugh Carter, who is on leave during the current academic year from the Department of Sociology of the University of Pennsylvania, and Miss Anna W. March.

Social Science Abstracts.—At the Hanover Conference of the Social Science Research Council, held during the latter part of August, it was decided that the first issue of *Social Science Abstracts* should appear in March, 1929. Only articles in periodicals will be abstracted in 1929, and books, monographs and serials will not be included until the second year. It is estimated, therefore, that about 15,000 articles will be abstracted the first year. From these beginnings *Social Science Abstracts* will work toward inclusiveness. The yearly subscription rate is $6.00, including annual indexes. Subscriptions and other communications may be sent to the Editor, 611 Fayerweather Hall, Columbia University, New York City.

PERSONAL NOTES

Professor Horace Secrist is on leave from Northwestern University for the current academic year, serving with Professor Frank A. Fetter of Princeton University as a special commission on economics at Claremont Colleges, Claremont, California. Part of their work during this year will be to carry out a piece of research of interest and value to business and professional interests in California, special emphasis being given to problems which are peculiar to the southern district.

Professor Warren M. Persons has resigned from Harvard University and is now associated with National Investors Corporation, 120 Broadway, New York City, as Vice-President.

Mr. Edgar Sydenstricker, who for the past thirteen years has been chief statistician of the United States Public Health Service and has had charge of its statistical investigations, has been made director of the new division of research of the Milbank Memorial Fund. Miss Dorothy Gerard Wiehl, also of the statistical staff of the United States Public Health Service, will be assistant director.

William Leonard Crum, Professor of Statistics, Graduate School of Business, Stanford University, has been elected editor of the *Review of Economic Statistics*, published by the Harvard Economic Society, Inc.

Dr. I. M. Rubinow, Executive Director of the Jewish Welfare Society of Philadelphia, has left that organization to go to New York City, where he will take up the work of the Zionist Organization of America.

Mr. Mortimer B. Lane has resigned his position as editor of the *Survey of Current Business*, Department of Commerce, and Mr. R. J. McFall, formerly of the Domestic Commerce Division, Bureau of Foreign and Domestic Commerce, has been appointed to succeed him.

Dr. Robert M. Woodbury has joined the editorial staff of the *Journal of the Social Science Abstracts* as associate editor.

Mr. Elmer J. Working, formerly member of the faculty of the University of Minnesota, has been appointed senior agricultural economist in the Bureau of Agricultural Economics, and will be engaged in analysis of grain prices.

Professor C. F. Remer of the University of Michigan is beginning work on a study of the international financial and economic relations of China. This study will be done under the auspices of the Social Science Research Council.

Dr. Charles A. R. Wardwell of the University of Pennsylvania has been appointed Assistant Professor of Economics and Acting Director of the Bureau of Business Research of Northwestern University.

Mr. Russell C. Engberg, formerly of the Institute of Economics, in April was appointed statistician-economist for the Federal Farm Loan Board.

Professor Paul R. Rider has been given a leave of absence by Washington University, St. Louis, for the academic year 1928–29, to accept a Sterling Research Fellowship in mathematical statistics at Yale University.

Mr. W. J. Carson, on October 1, resigned his position with the Division of Research and Statistics of the Federal Reserve Board, Washington, D. C., to accept an appointment as Assistant Professor of Finance at the Wharton School of Finance and Commerce, University of Pennsylvania.

Dr. William Harvey Young has joined the Regional Commercial Survey Section of the United States Department of Commerce.

Mr. H. S. Davis, formerly research assistant at Denver and last year engaged in research in the wool industry at the University of Pennsylvania, where he also served as instructor, has joined the Bureau of Business Research and the Department of Statistics at the University of Pittsburgh.

Mr. Benjamin P. Chambers, formerly of the Reports Department of the Federal Reserve Bank, on August 15 accepted a position with the Statistical Department of the National Bank of Commerce in New York.

Dr. Victor S. Clark has recently joined the staff of the Institute of Economics, where he and certain other members of the staff will engage in a study of economic conditions in Porto Rico.

Mr. Leo Pasvolsky of the Institute of Economics is spending a year in Europe, engaged in a study of economic conditions in the Danubian states.

MEMBERS ADDED SINCE SEPTEMBER, 1928

Baker, Oliver P., Detroit Board of Commerce, 320 West Lafayette Boulevard, Detroit, Mich.

Battles, Ralph U., Department of Rural Economics, Ohio State University, Columbus, Ohio.

Bayly, Richard C., E. E. MacCrone and Company, Penobscot Building, Detroit, Mich.

Blood, Kenneth D., Bureau of Agricultural Economics, United States Department of Agriculture, Austin, Tex.

Booz, Edwin G., Edwin G. Booz Surveys, 14 East Jackson Boulevard, Chicago, Ill.

Bradford, Dr. Frederick A., College of Business Administration, Lehigh University, Bethlehem, Pa.

Brown, Frederick W., The National Committee for Mental Hygiene, 370 Seventh Avenue, New York City.

Congdon, Professor Allan R., University of Nebraska, Lincoln, Neb.
Corrin, A. Sturgis, 212 West Washington Street, Chicago, Ill.
Courtright, W. D., Merchants National Company, 660 South Spring Street, Los Angeles, Cal.
Cowan, Robert G., Research Division of the Federal Reserve Bank of New York, 33 Liberty Street, New York City.
Crowell, E. C., Statistical Department, Bell Telephone Laboratories, 463 West Street, New York City.
Crowther, Ernest, Dean of Pittsburgh School of Accountancy, Pittsburgh, Pa.
Crutchfield, James S., President, American Fruit Growers, Incorporated, 2100 Penn Avenue, Pittsburgh, Pa.
Dayton, Dr. Neil A., Department of Mental Diseases, State House, Boston, Mass.
Duff, William M., The Edward A. Woods Company, General Agent, The Equitable Life Assurance Society of the United States, Pittsburgh, Pa.
Duncan, Walter G., Swift and Company, Union Stock Yards, Chicago, Ill.
Ekin, John J., The Baltimore and Ohio Railroad Company, Baltimore, Md.
Enders, Mary P., Bureau of Statistics and Information, New York State Department of Labor, Albany, N. Y.
Fluer, Louis, West Penn Power Company, 14 Wood Street, Pittsburgh, Pa.
Frecheville, George, Agricultural Economics Research Institute, University of Oxford, England.
Freeman, Allan R., Standard Statistics Company, 200 Varick Street, New York City.
Folger, D. Campbell, Western Electric Company, 195 Broadway, New York City.
Fowler, E. J., Commonwealth Edison Company, 72 West Adams Street, Chicago, Ill.
Hall, Orville J., Department of Rural Economics and Sociology, University of Arkansas, Fayetteville, Ark.
Heald, Kenneth C., Gulf Oil Companies, Pittsburgh, Pa.
Hoehn, George W., National City Bank, 42nd Street and Madison Avenue, New York City.
Hudson, Albert M., 6439 Baltimore Avenue, Kansas City, Mo.
Humiston, Burr H., McGill Commodity Service, Taylor Building, Auburndale, Mass.
Leech, Lewis Harper, Editorial Department, Chicago Tribune, Chicago Ill.
Lewis, Mabel S., 10 Jones Avenue, Chatham, N. Y.
Ligo, Clyde F., Statistician, Pittsburgh Railway Company, 435 Sixth Avenue, Pittsburgh, Pa.
Lohr, Allen W., Westinghouse Air Brake Company, Wilmerding, Pa.
Main, Frank W., Main and Company, Farmers Bank Building, Pittsburgh, Pa.
Melzer, Dorrit I., Student, Columbia University, New York City.
Moscoso, Professor Tobias, University of Rio de Janeiro, Brazil, S. A.
Murray, Nat C., 231 South La Salle Street, Chicago, Ill.
Neal, Robert W., Pacific Underwriter and Banker, 417 Montgomery Street, San Francisco, Cal.
Nekrassoff, Col. Vladimir A., Bell Telephone Laboratories, 463 West Street, New York City.
Nevin, J. C., Managing Director, Pittsburgh Branch, Federal Reserve Bank of Cleveland, Pittsburgh, Pa.
Newman, Paul C., United States Department of Agriculture, Grand Forks, North Dak.

Nicholson, J. H., Vice-President, National Tube Company, 1824 Frick Building, Pittsburgh, Pa.

Nixon, James W., International Labour Office, League of Nations, Geneva, Switzerland.

Nymeyer, Frederick, Armour and Company, Union Stock Yards, Chicago, Ill.

Ordan, Louis, American Bureau for Financial Research, 225 Broadway, New York City.

Ottman, Frederick R., Instructor, Department of Economics, School of Commerce, New York University, New York City.

Oxenreiter, A. E., Joseph Horne Company, Pittsburgh, Pa.

Rheinstrom, C., 1926 Broadway, New York City.

Robinson, Alexander C., President, Peoples Savings and Trust Company, Wood Street and Fourth Avenue, Pittsburgh, Pa.

Roche, John J., 355 Broadway, Milwaukee, Wis.

Seabury, Roland T., Rust Craft Publishing Company, 1000 Washington Street, Boston, Mass.

Schram, Jennie L., Research Department, Illinois Chamber of Commerce, 35 East Wacker Drive, Chicago, Ill.

Schware, Pearl, Forbes Magazine, 120 Fifth Avenue, New York City.

Schwarz, P. Manfred, Corn, Schwarz and Company, 15 William Street, New York City.

Sias, Professor Azariah B., Ohio University, Athens, Ohio.

Spicer, Hazel I., Children's Bureau, Department of Labor, Washington, D. C.

Strickland, E. Franklin, Heppenstall Forge Company, 95 Howard Avenue, Bridgeport, Conn.

Thompson, Professor J. Murray, Lehigh University, Bethlehem, Pa.

Toan, Winthrop, Columbia University, New York City.

Tyler, Dr. Ralph W., University of North Carolina, Chapel Hill, N. C.

Ulrey, O., Department of Economics, Michigan State College of Agriculture, East Lansing, Mich.

Ward, Norman B., Peoples Savings and Trust Company of Pittsburgh, 307 Fourth Avenue, Pittsburgh, Pa.

Wardwell, Dr. Charles A. R., Northwestern University, Chicago, Ill.

Wiley, Dr. Clarence A., Department of Economics, University of Texas, Austin, Tex.

REVIEWS

Health and Wealth, by Louis I. Dublin. New York: Harper and Brothers. 1928. 360 pp.

Under this happy alliterative title Dr. Louis Dublin has given us in fifteen chapters a study of the economics of national and international health, every page of which deserves to be carefully read by every social worker or well-wisher on both sides of the Atlantic, whether medical or non-medical. Happily, the reading of these chapters cannot be regarded as a task to be incurred; for each problem is discussed in a most readable fashion, and one does not need to regard statistics as did a great English Chancellor of the Exchequer, Mr. Goschen, as more fascinating than an exciting novel, to make one glad to have such an attractive presentment of our chief social problems and their remedies, as is given in this volume.

Economics often form a dreary study to the uninitiated; but when touched with the fervor of desire after and aim at reform, economics and their statistical statement become a "live" thing. In his volume Dr. Louis Dublin shows that he shares the burning zeal of William Farr, the father of English vital statistics, who throughout his long official career year by year made his statistics a means for furthering needed public health reforms.

In the first three chapters Dr. Dublin discusses the economics of world health, the cost of medical service, and the cost of neglected children. Then follow chapters on the problem of heart diseases, of tuberculosis, of cancer and of old age. The constitution of the American population, and the falling rate of natural increase of population, birth-control, home-making, the Negro, industrial mortality, prohibition, and possible extensions of human life, complete the list of subjects discussed.

All that can be attempted here is to sample some of the items of information. It is noteworthy, for instance, as showing the extent to which public or semi-public treatment of disease has increased, that about ten millions of the American population are receiving clinic care. In New York City alone about a million patients make six million clinic visits annually. Dr. Dublin advocates that free service in hospitals should be gradually eliminated, and the present system of graduated payment extended. In discussing heart diseases he rightly emphasizes the need for attacking the earlier diseases, *e. g.*, rheumatic fever and syphilis, which cause later disabling cardiac disease, and not limiting attention to the later degenerative changes of old age or the bad habit of middle life.

The chapter on tuberculosis is judicial and balanced, as between innate qualities and the results of bad conditions of living. It is well to have brought home very clearly the supremely important rôle of industrial life in provoking tuberculosis; and the hopeful suggestion with which this chapter ends—that "tuberculosis must either disappear or become a relatively rare disease"—

may be commended for reflection by those who still remain lukewarm or even icy-cold as to the value of preventive measures against this disease.

It would be easy to cull extracts from each later chapter in this extremely suggestive volume, which would emphasize the desirability of its wide circulation. But what has been written will suffice to indicate that Dr. Dublin is a sound and cautious guide, through the maze of our social vital statistics, but that at the same time he is so fully hygienically-minded, that all engaged in public health and other branches of social work may expect with confidence to derive inspiration from these pages.

ARTHUR NEWSHOLME

The Behavior of Prices, by Frederick C. Mills. New York: National Bureau of Economic Research, Inc. 1927. 598 pp.

This "notable volume"—rightly so appraised by the Directors of Research in their foreword—is to be considered as "part of a general attack upon the problem of charting the price system, defining its elements, tracing the connections between these elements, determining the nature of the changes which occur in the price system with the passage of time and with changes in general economic conditions, and of describing more exactly the part which the system of prices plays in economic processes." In the completed portion of his investigation Dr. Mills probed the general problem: He selected the characteristics of commodity prices which could satisfactorily be measured, made the measurements, classified the results, and noted whatever uniformities appeared. Interpretation and synthesis were deferred, however, to a later study which would undertake to define some component elements in the price system and to describe their interrelations.

The Behavior of Prices is addressed to the "attentive reader," but that "distinguished person" should be—and will be—either the economist capable of interpreting the results or the statistician able to appraise the technique. Not that anything has been slighted in the tabular, graphic or verbal presentation to make the conclusions stand out clearly. The book is a model in these respects. But it is the account of a tremendous statistical broadside with the targets not designated, and the abundance and variety of results are confusing in the absence of unifying theses.

The book is comprised of four clearly separable but interrelated sections:

Chapter I—Measurable characteristics of commodity prices.

Chapter II—Regional differences in commodity prices and in price behavior.

Chapter III—The measurement of price instability: price relatives in combination.

Chapter IV—Measures of price behavior in combination.

In addition to the numerous tables and graphs in the text the appendix contains an invaluable description of the quotations of 457 wholesale prices published by the Bureau of Labor Statistics and some one hundred pages of tabulated statistical results.

The characteristics of individual commodity prices to be measured in Chapter I must "possess economic interest and lend themselves to quantitative treatment."

Those selected are (1) change between specific dates, (2) variability, (3) trend, (4) duration, timing and amplitude of individual price changes in the business cycle, (5) flexibility: correlation between prices and quantities. The reader cannot proceed far in this section without remarking on the resourcefulness of the investigator; for the measures of variability and frequency of price change are ingenious, and the analysis of individual price changes in the business cycle is noteworthy for its simplicity and effectiveness.

Chapter II suffers the handicap of extreme paucity of data. While the other chapters are almost entirely confined to wholesale commodity prices by the lack of information on prices at retail or prices of services, the regional analysis must be still further restricted to those few series with parallel quotations in different markets. The treatment of the problem can, therefore, be little more than an illustration. But the handling of regional differences in prices seems to the reviewer to labor under a second handicap—that of a device ill adapted to the problem. The mean deviation of regional prices does reflect the extent of price differences, but it fails to lead toward an explanation of these differences. In the second portion of the chapter the importance of regional differences in the *behavior* of prices is well illustrated.

To many statisticians and economists the analysis of price instability will be the most significant portion of the book. Four types of instability are recognized, change in the average level of prices, dispersion of price relatives about an average, shifting in position (rank) of individual price relatives, and "instability" of populations of price relatives. The illustrations of bias in index numbers, the attempt to apply the theory of sampling in measuring their unreliability, and the use of frequency distributions in their computation are among the technical by-products of measuring stability of the price level. More interesting is the index of dispersion of price relatives as a device for gauging the unrepresentativeness of index numbers or, from another viewpoint, the magnitude of the "disruptive forces" which bring about unequal price changes. This section is a strong argument for supplementing the usual price index with a measure of dispersion. The case for the index of displacement is not so obvious. Certainly the two indexes afford much the same information in the illustration on pages 287–8 if the year of reference for the index of displacement, 1914, be taken also as the year of reference (base year) for the index of dispersion. The index of displacement, $1-\rho$, in which ρ is the coefficient of rank correlation between relatives in the year of reference and the given year, measures the shifting of individual relatives, *i. e.*, changes of rank. But the index of dispersion also measures significant shifting of individual relatives, namely, the movement away from the average since the base year. Finally, the index of displacement is difficult to interpret, depending, as it does, on prices in three years, the base year, the year of reference, and the given year.

Most novel of the measures of instability are those dealing with the population of prices. The order of the lowest theoretical moment becoming infinite is taken as a criterion of the stability of the distribution. Low order infinite moments with location of the β_1, β_2 point in the heterotypic region of Rhind's diagram are commonly encountered, indicating the sporadic play of unbalanced forces

or, perhaps, heterogeneity of the data. Nearly all the distributions depart widely from normal, with type IV predominating. The full significance of this approach to the stability of price populations is not yet apparent and further research would seem to be indicated.

The last chapter reviews the characteristics of behavior of individual prices in combination, that is, grouped in frequency distributions. Here additional light is shed on uniformities and diversities in the movements of individual prices. For students of the business cycle this chapter is of special interest.

It is beyond the scope of a review to deal adequately with the numerous specific findings of the investigation. They may be read in the excellent summaries at the end of each chapter, or better still in the main body of the report. Let it be sufficient to say here that many uniformities of price behavior are discovered, that certain patterns do reoccur partially in business cycles, and that successive stages of these cycles are linked together quite definitely. But the residual unexplained variation is almost always very large. Not that it should be inferred that there is little of definitive value in the work; it is filled with data relating to problems on which we have long wanted—and lacked—specific data.

It is no disparagement of the present report that a fair appraisal of its merits must be deferred until the publication of the second volume. In that study the respective rôles to be assigned to price data and to other determining factors in explaining the price system will have particular interest for many economists. All of us await the completion of what promises to be one of the foremost statistical contributions to economic knowledge in this decade.

THEODORE O. YNTEMA

University of Chicago

Advertising Fluctuations, Seasonal and Cyclical, by William Leonard Crum. Chicago: A. W. Shaw Company. 1927. xxvii, 308 pp.

To our knowledge of the business cycle and its manifestations in specific lines of activity this study of the cyclical and seasonal movements of advertising space in newspapers and magazines is a welcome addition. The general outlines of the attack upon the problem are the same as those of *The Iron Industry in Prosperity and Depression*, issued sometime previously by Professor Crum in collaboration with Professor Vanderblue.

The data used in the study were, for the most part, drawn from *Printer's Ink* and the *New York Evening Post*. Since the measures of advertising activity are in terms of agate lines the series represent volume of advertising, and, lacking the price factor, one element of complication in the analysis of the data is absent. The presence of marked seasonal variations tending to overshadow the cyclical movements renders the elimination of seasonal variations the most important statistical problem. This is preceded by a proper adjustment for the number of weekdays and Sundays per month and, in the case of weekly magazines, for the number of issues per month. The problem of adjusting for the number of Sundays per month in the case of newspaper advertising, when some papers issue Sunday editions and some do not, is solved in a simple and ingenious manner.

No attempt is made to eliminate the secular trend of advertisements, for "it is not thought that this element of the total fluctuation is sufficiently important in most of these series to justify its elimination, and the presence of trend in the data after correction for seasonal variation does not in most of the series preclude a thorough recognition and interpretation of the cyclical fluctuations themselves." The irregular movements, Professor Crum concludes, are relatively few and unimportant.

Monthly data upon a national basis are examined for magazine space since the War and for newspaper space since 1921. Quarterly data are considered for magazine and newspaper advertisements since 1907. Largely because much of the advertising in newspapers is local in nature and thus represents local conditions, the author believes: "Advertising fluctuations in magazines have a somewhat more general bearing upon the problem of variable advertising effort than do corresponding fluctuations in newspaper advertising."

Data upon a regional basis are studied for newspaper space in a large number of leading cities of the country—for the most part treating of the years 1921 to 1926.

Data for New York newspaper advertising space, by lines advertised, are carried back usually to 1916; the specific lines considered were real estate, automobiles, clothing, finance, "wants," and recreation. Many of these series of New York newspaper advertisements are compared with appropriate indicators of business movements but it is to be regretted that with the exception of one curve (page 10) comparing total newspaper advertising lineage in the United States and bank debits outside New York, and another (page 11) showing total magazine advertisement lineage in the United States and bank debits outside New York, there are no comparisons of cyclical fluctuations in advertisements on a national basis with those of other series for the country as a whole. Concerning these two comparisons, Professor Crum says: "Through most of the movement since 1919, there is a clearly noticeable tendency for the fluctuations in the advertising curves to precede slightly corresponding movements in the curve of general business activity. This anticipatory character of cyclical fluctuations in advertising is to be expected in view of the obvious relationship between advertising activity and the business which it serves to produce."

One of the pleasing features of the book is the almost exclusive use of charts with logarithmic vertical scales.

FREDERICK E. CROXTON

Columbia University

Urban Land Economics, by Herbert B. Dorau and Albert G. Hinman. New York: The Macmillan Company. 1928. xvi, 570 pp.

Urban Land Economics, by Dorau and Hinman, is the latest addition to the textbook material in the new field of land economics. Teachers of the subject will welcome it with open arms, as it will greatly simplify the problem of finding suitable reading assignments for their growing classes. The book is written clearly and interestingly, and will doubtless remain the standard text in urban land economics for some time to come.

In view of the newness of the field and the certainty that this book will influence

its future development, it is interesting to note what the authors conceive to be the scope of the field. They have divided the book into five parts. Part I deals with the economic basis of city development, the location of cities and their internal structure; Part II with urban land utilization; Part III with the various ways in which these uses are controlled by public authority, such as city planning, zoning, rent regulation and taxation; Part IV is concerned with urban land tenure and tenancy; while the interesting theoretical problem of land income and land value is reserved to Part V. Finally there is a concluding chapter in the nature of an appendix in which the authors attempt the dangerous but fascinating task of predicting the future of our cities.

It was a mistake, I think, to put the chapters on land income and land value at the end of the book. As a result the authors are frequently forced to anticipate their doctrine of rent and value. This is notably true of the very interesting discussion of the production costs in land utilization in Part II, and of taxation in Part III. It would be better to combine the chapter on the nature of urban land in Part II with the chapters on land income and land value and insert them immediately following the introductory chapters on the development of cities.

Part V is the most interesting and provocative part of the book. The authors vigorously defend the thesis that the private receipt of rent is not only necessary to the best utilization of land, but also that it is largely earned. "On the average a large proportion of the rent of land is payment for desirable economic activity performed in the past in anticipation of rent or in anticipation of an increase in rent" (page 494). "Rent as an economic reward can be justified in the same way and to the same extent that interest, wages, or profits are justified" (page 492). Those who are interested in this question will find here an able defense of one side of this much disputed point. Certainly it is true that the nature of many of the investments in land, and particularly in urban land, forces men to push beyond the margins of cultivation. They must anticipate the future and secure the competitive return in the subsequent increase in rent or in land value. For the best utilization of our land resources some part of the future increase in rent is thus undoubtedly earned. But it is equally true that the lure of the increment has also caused incalculable wastes by inducing men *unnecessarily* to push cultivation beyond the margins, thus bringing about an unwise apportionment of the factors of production. This side of the question deserves more emphasis not only in the discussion of land income but also in the discussion of land taxation.

Whatever his views on these questions of principle the teacher will find the book an excellent basis for class room discussion. It is a needed and valuable addition to the literature of the field.

JOHN V. VAN SICKLE

Social Science Research Council

Statistique Mathematique, by Georges Darmois. Paris: Gaston Doin. 1928. xxiv, 363 pp.

While French authors have frequently written books on the theory of probability with a chapter devoted to its application to statistics, to find a whole

work in the French language on statistical methods or the mathematics of statistics is very rare. The recent work of Jordan, published in 1927, with the present book of Darmois will enable the French statistician to find in his own language scholarly treatises on mathematical statistics.

In 1922 an Institute of Statistics was founded at the University of Paris. For the first two years the courses in the theory of probability and its application to statistics were given by Emile Borel. He was followed during 1924–25 by Darmois, and the present book owes its start to those lectures. As there was already at the Institute an elementary course in the methods of statistical practice, Darmois devotes himself to an exposition of the underlying mathematical theory and presumes his readers to be familiar with the differential and integral calculus.

The book, bound in paper, is clearly printed. Of the 363 pages, the last 60 are devoted to tables, notes of a mathematical character, a bibliography, an index, and a table of contents. The preface is written by Michel Huber, Directeur de la Statisque Générale de la France. The book is well planned and follows along logically. It contains the most important topics—some with more detail than others—and includes recent advances in mathematical statistics. The author shows his wide acquaintance with the field by selecting from various sources those proofs of theorems which are most simple and elegant. References to other writers and sources enable him to leave out in places cumbersome algebra and long, involved proofs. It is not a book devoted alone to mathematical theories and demonstrations; the author points out the danger of being deceived by mathematical rigor into making conclusions which are not justified by the data. Thus on page 303—"*nous allons en donner des exemples, qui montrent bien, comme de dit Charlier, que la statistique mathématique n'est pas un appareil automatique, dans lequel on placerait à une extrémité le matériel d'observation, pour recueillir à l'autre extrémité la solution exacte du problème.*"

The reviewer was particularly impressed with the first three chapters which present the fundamental laws in the theory of probability. Examples from urn schemata or games of chance illustrate the theorems. Then mathematical expectation is defined, and with its aid, formulae for moments, means and deviations derived. The inequality of Tchebycheff gives a simple proof of the theorem of Bernoulli. The "*fonction caracteristique*" introduced by Cauchy in the theory of probability in 1853, and later used by Poincaré, proves a powerful weapon in attacking later theorems. The normal law of frequency, although not derived till Chapter 10, is studied with the aid of this function and its moments calculated. Chapter 3 is devoted to a study of the theorem of Bernoulli and its developments, the series of Bernoulli and Poisson, and the law of small probabilities, due also to Poisson, sometimes called "the law of small numbers."

Chapters 4 and 5 deal with frequency functions of one variable, represented graphically by a polygon or a continuous frequency curve. The author gives the derivation of the differential equation from which Pearson obtains his seven types of frequency curves. He then lists the seven types and shows how the constants in the equations of types I, II, and III can be determined by the method of moments. For a fuller discussion we are referred to Elderton.

Pearson's method for dissection of a compound curve into two components is also lightly treated. Then follows Charlier's method of developing a continuous frequency distribution in a series of orthogonal functions. The validity of this development is discussed as well as some recent work by Henderson and Romanovsky. The author ends with Charlier's expansion for a discontinuous distribution (the reference on page 141 should read Chapter III). In Chapter 5 we have the coefficient of dispension of Lexis, the coefficient of divergency of Dormoy, the coefficient of instability of Charlier, the test of goodness of fit of Pearson, the series of Lexis and several problems in the theory of sampling. To the reviewer the discussion on frequency curves is inadequate. It is doubtful if the ordinary reader could bridge the gap in developing the theory of Pearson and obtain a working knowledge of the subject. A comparison of the two methods of curve-fitting on the same data would enlighten the reader on the amount of computation involved. No mention is made of the possibility of negative frequencies when using a Charlier series. The method of moments is explained, but the method of least squares supposed known.

The importance of the theory of correlation and its use in so many varied fields requires the author to devote the remainder of the book—slightly less than one-half—to this subject. Chapters 6–9, inclusive, treat of those matters which are found in Yule, but the notation and proofs follow Tschuprow. In Chapter 10 the normal law is derived from the standpoint of elementary errors; multiple and partial correlation discussed for three variables. Chapter 11 deals with the correlation of time series—the author uses the term "covariation" due to March. Various methods of eliminating the time element are discussed, some work of Anderson explained; the papers of Yule, Persons and others are listed for reference at the end of the chapter, but are not analyzed.

There is no chapter devoted to the theory of sampling. The standard error of the pth moment about a mean is developed in Chapter 5; that of mean square contingency, regression coefficients, and correlation ratio are found in Chapter 9. Perhaps a complete discussion of this topic was beyond the intention of the author.

J. R. Musselman

College for Women, Western Reserve University

An Investigation of Economic Data for Major Cycles, by Charles A. R. Wardwell. University of Pennsylvania. 1927. 147 pp. plus tables.

Dr. Wardwell offers the suggestion that the well known trio of trend, season and cycle, whose themes have been entrancing the students of business cycles, especially since the War, might even spell greater magic to the statistician if the trio were changed to a quartet by the addition of his newly discovered major cycle. The sympathetic critic would like to know, however, whether the major cycle has those qualities which mark a distinct individuality and, secondly, whether its performance does really enrich the ensemble as is claimed. Let us see how Dr. Wardwell attempts to prove his case.

By computing one-year moving averages of quarterly figures for ten American, three English and four German series, the seasonal element is eliminated. To

isolate the cycle, arithmetic averages of the items intervening between peaks and troughs determined by inspection are calculated. These averages of cycles, computed from peak to peak and trough to trough, are centered and connected by an arithmetic interpolation of the quarterly figures. These "cyclical-moving averages," called the "intermediate trend," which are used as the basis for eliminating the business cycle, are undoubtedly superior to the moving averages of a fixed number of items, for they are not chained to an arbitrary time limit, but vary with the length of the cycle. The author, however, is not content with this happy device for isolating the "growth factor," for he proceeds to fit in most cases a second or third degree parabola by the method of least squares to the excellent trends already established. The resulting deviations are the major cycles. All of the seventeen series, it is claimed, show evidence of at least one major cycle, although two or three cases of this phenomenon are most common.

A comparison of the timing, amplitudes, and duration of these swings in the seventeen series gives us the quintessence of the major cycle. For the United States 1890 marked its peak, 1895 the trough, 1906 the next peak, 1914 the second trough and 1918 the last peak. The duration was about 15 years, a figure obtained by averaging the length of time it took for these "major" fluctuations to run their course, although it should be noted that the average is made up of a range that shows the pig-iron series to have a duration of almost 9 years, and wholesale prices an average length of a little over 19 years. In Germany the crests of these major swings were noted in 1890 and 1898, while the troughs were reached in 1892 and 1903. The duration was 9.5 years. For England the evidence was inconclusive. Furthermore, with the exception of the reversal of positions between stock prices and pig-iron production, the major cycles show a time-sequence that is similar to that which the ordinary cycles are supposed to reveal.

The core of the explanation for these long-time swings, we are told, lies in the over expansion of the fixed assets of business establishments. For the purpose of development of the argument, the discussion assumes that the typical major cycle contains three of the shorter ones. If we start with the end of a very severe period of depression, the business cycle that follows will be a rather mild one because the weakest organizations will have been eliminated in the hard times that had previously struck industry, and memories of past errors on the part of the survivors will be quite vivid. The second business cycle will have more violent characteristics. Memories will have correspondingly weakened with the passage of time; new firms will have come into the field and the volume of business, especially of producers' goods, will have increased considerably with the vigorous competition for immediate deliveries of equipment to new as well as old firms. With the turn of events for the worse, there will be widespread a downward revaluation of current assets, but there will be great resistance to any tendency of scaling down what has proved to be an excessive investment in fixed assets. The burden of bearing these costs will be carried into the next cycle which will be denoted by a relatively mild period of revival and a rather severe period of depression. By this time the "secular" growth of industry will have caught up with its over-expanded equipment. In brief, then, the

major cycle is the result of an inheritance which is bequeathed by one business cycle to another; up to a certain point the inheritance becomes progressively larger as it is passed on by each succeeding cycle, and beyond that point it becomes cumulatively smaller with the passage of time—that heritage is the "pecuniary investment in fixed plant and equipment."

To the reviewer it seems that the author's effort to establish the individuality of the long-time cycle falls short on both empirical and theoretical grounds. Are not these major-cyclical deviations the result of the process of curve-fitting by the method of least squares to the moving averages of the cycles? It is difficult to see why the author has seen fit to publish all the preliminary calculations but has failed, with one exception, to include the figures of secular trends and major cycles. We are thus unable to study the behavior of the actual data as they passed through the final stages of manipulation. The only recourse we have is to look at the charts which were unfortunately marred in many cases by the process of reduction, so that numbers on the abscissae and ordinates are intelligible with great difficulty. Careful examination of these graphs leads the reviewer to conclude that most of the series yield the results recorded because of the type of curves employed. With very few exceptions, comparisons of the charts showing the secular trends and intermediate trends (*i. e.* the lines connecting the averages of the cycles) point to the inference that the deviations were due to differences between the slopes of the intermediate and secular trends; or put a little differently, the amplitudes of the major cycles are really the measures of the magnitude of error at various points between the slopes of these two lines.

Nor is the hypothesis suggested in support of major cycles much more convincing. Does not the author place too heavy a responsibility upon capital-producing industries for bringing about the course of these major swings? We are not clearly shown why there should be such great discrepancy between the rewriting of current assets and fixed assets in the downward movement of the cycle. There is a wide gap between the theory that is developed and the statistical data presented, whereas theory and fact might have been interestingly confined within communicable limits, if the author had seen fit to weigh his theory in the light of the detailed business events as recorded by such trade journals as the *Iron Age*, or in the light of such materials on specific industries gathered by the old Bureau of Corporations. The only link between hypothesis and the data that is mentioned by the author is the case of the high and low points of pig-iron production preceding those of any other series. "This seems to enforce the point that the swings of the major cycle are in the main swings in the production of fixed capital" (page 133)—a statement which does not largely connect with an argument that runs in terms of pecuniary revaluation. Nor are the mortality-rates and birth-rates in the number of firms nearly so important as the behavior of the surviving firms. Furthermore, the argument premises a major cycle of psychological moods in which the spirit of recklessness or caution becomes cumulative over a considerable period of years—a postulate which should be accepted with a healthy dose of skepticism.

ASHER ACHINSTEIN

Washington, D. C.

Health Problems in Organized Society—Studies in Social Aspects of Public Health, by Sir Arthur Newsholme, K.C.B., M.D. London: P. S. King and Son, Ltd. 1927. xi, 253 pp.

In this collection of addresses and lectures delivered during the past few years at various American universities and in England, Sir Arthur Newsholme has discussed a large variety of the social aspects of public health. Without, perhaps, deliberate intention, he has revealed his philosophy of social activities and public control in the maintenance of the health of the population, a philosophy which has developed out of an unusual experience enriched by forty-five years of public service. In this period of nearly half a century as a medical officer of health, as a scientific investigator, and as a writer, he has been in the forefront of the progressive wing of the army of sanitarians in the sense that he has stood for a greater measure of public control in the affairs of the people which relate to their health.

In this volume, however, he reveals his position on certain social aspects of his subject which, although progressive in the sense already referred to, is at variance with the views of a group who at the present time constitute one extreme wing on these matters. Perhaps an intimation of this attitude is given most clearly in his lecture on "The Growth of the Preventive Idea in Medicine," given at the University of Michigan in 1926, when he referred to two aspects of preventive medicine which he regards as being extremely important. "The first of these," he said, "is the increasing realization of the intimate relation between success in preventive medicine and in the control of our chief social evils." It is significant to note that he includes among these social evils not only such problems as housing, education, recreation and poverty and destitution, but gambling, alcoholism, crime and delinquency. He regards the control of these evils not only as necessary to public health but as a practicable procedure in that "the true method of approach to their solution is by the *case method*" since "each individual, when aberrant from the normal, is a problem needing individual investigation and possible causal factors need dissecting out from the tangle in which they are usually found, with a view to the discovery of the most potent, and suitable application of curative and preventive measures."

He goes further, however, in the second aspect which he regards as so important when he lays emphasis upon "character" as "a supremely important factor in securing success in preventive medicine." This theme runs through the entire book and is clearly evident when he discusses the social diseases, particularly venereal diseases, and alcoholism. Sir Arthur may be accused by some as being old-fashioned in his point of view, but he supports his text with undoubted sincerity and even fervor. "Does anyone doubt," he asks, "that if public health authorities, voluntary social workers, educators, employers, patients themselves and their relatives, were all possessed of the knowledge which hygienists possess, and were ethically impelled without exception to act consistently and persistently on that knowledge, tuberculosis in another generation instead of being, like venereal disease, one of the four or five greatest scourges of mankind, would assume a relatively unimportant place among human enemies? If this statement

is correct, the moral and spiritual attitude constitutes a chief hope for the future; and this attitude is only possible if cultivated."

In these days when the tendency is to discuss venereal diseases, limitation of size of families, prohibition, and other like questions from a purely technical and almost mechanical point of view, the appeal to what some might term the old moralities by so distinguished an authority will be refreshing to many.

It would be unfair, however, to characterize Sir Arthur's collection of studies on health problems in organized society as a moral screed; such a characterization would be erroneous and grossly misleading. The volume is filled with the results of a wide and ripe experience in progressive public health work. The broad range of Sir Arthur's activities, his painstakingly scientific point of view on all technical questions, and his practical-mindedness are everywhere evident. His criticism of the health insurance in England is of particular interest to this reviewer since one of his recommendations is almost identical with the view put forward over a decade ago in an official publication of the federal Public Health Service by the reviewer and Dr. B. S. Warren. This view is aptly expressed in Sir Arthur's own words when he says: "When preventive medicine and insurance are indissolubly married then we may anticipate a striking advance in the general standard of health and in the average duration of life." To accomplish this, as he points out, it will be necessary for every practicing physician to be a medical officer of health. If thereby the physician will develop into a practitioner of preventive medicine as well as an expert in the relief of individuals who have become ill, then by all means let us violate traditional ideas about insurance or medical practice or any other detail involved.

EDGAR SYDENSTRICKER

Milbank Memorial Fund

Standard Departmental Stock-Sales Ratios for Department Stores, by Carl N. Schmalz. Michigan Business Studies, Volume I, No. 4, 1928. 62 pp.

Sales Quota Systems, by C. E. Griffen. Michigan Business Studies, Volume I, No. 5, 1928. 43 pp.

The study by Mr. Schmalz is the fourth of a series of studies published by the Bureau of Business Research, School of Business Administration, University of Michigan. It applies to data for 52 department stores during the spring months of 1927, its purpose being to devise a method of establishing standards regarding the relationship between stocks and sales in representative departments, and in all departments of these stores.

The author finds that for purposes of store administration, the conventional stock turn figures are inadequate inasmuch as they are averages based on past performance—generally a year. What is wanted is a standard for prospective stocks. This the author finds by fitting a curve—defined by the equation, $Y=aX^b$, when Y is the normal stock-sales ratios corresponding to any given volume of sales, X—to the ratios of stocks on hand at a beginning of a period (month) to the sales during this period.

When these ratios for departments by months, and for all departments for all

months, are plotted on charts, it is found that the relationship is systematic and capable of being defined by the equation given above. Curves are fitted to the data, and standard ratios derived from them. These are published in an appendix, for the entire stores and for fourteen departments for the months February to July, inclusive, 1927.

Considerable is said about the meaning which merchants should attach to these standards, and the ways in which they may be used in merchandising control. Sufficient emphasis, however, is probably not given to the fact that they are in the nature of *averages*, that merchants may use them to determine not necessarily the most desirable size of stock, but the extent to which their own stock-sales ratios vary from the average—a figure defined by the terms of the equation which "fits" the data. If such knowledge is of value for administrative purposes, then the standards defined are significant. If, however, because of peculiar location, seasonal trade, and customer demand, variation from the average is desirable, then the figures are standard only in the sense that they are average, and not in the sense that they should be followed. It should be noted that variable sales volumes refer to *different* stores, and not to the same store under varying conditions—a distinction too frequently ignored in discussions of the relationship between store operation and sales volume.

This study, essentially statistical, breaks new ground, and should be of service to the trade with which it deals.

Professor Griffin's study is non-statistical, although some attention is given to the subjects of limitation to market measurements, and market value indices. For the most part, it is descriptive, having for its general purpose, as the author says, "to survey the existing practice in setting sales quotas and to consider the usefulness of the different methods and their adaptability to different objects."

HORACE SECRIST

Northwestern University

Labor Conditions in Japan, by Schuichi Harada. New York: Columbia University Press. 1928. 293 pp.

This study approximates the scope and comprehensiveness of a yearbook. The author interprets his subject broadly and, rightly no doubt, assumes that the welfare of the laboring class is fast bound up with almost every other phase of the economic and social life of the Japanese people. Hence he gives over the first third of his book to a statistical discussion of the industrial development of Japan, its natural resources, agriculture and food supply, investments and trade (foreign and domestic), and the growth and movement of population. The tables presented are evidently derived, for the most part, without laborious computations from the reports of governmental bureaus and departments. But they are the more welcome because many of these sources are not readily accessible to the general reader in America. Incidentally, in this portion and in his later discussion Dr. Harada has occasion to lament the inadequacy or extreme recentness of Japanese figures on several matters—the amount of capital invested

in different industries, the proportion of workers producing for export, unemployment up to 1925, and the cost of living prior to 1922.

Scarcely so much space was needed, however, to show the unequal and unbalanced state of Japanese economic development. Owing largely to the admittedly abject imitation of foreign methods, to the artificial hastening of the process by the government, and to the over-expansion which took place everywhere during the War, things are, indeed, rather sadly out of line in Japan; and the result is a discontent and restlessness that affects not only the working people at home but is fraught with danger both for occidental trade and for the peace of the world. In spite of the fact that agriculture is the major occupation of nearly half of the families and of more than half of the population, Japan is compelled to import a large and evidently increasing portion of her rice and other foodstuffs. With wages at from a sixth to a fifth of what they are in the United States, the purchasing power of the average family is so small and the prevailing standard of living so low or so different as not to furnish a basis for large-scale machine production, and a considerable proportion of the output of the new factories must seek a foreign market with all of the attendant uncertainties as to competition, substitution, and tariffs. In fact, where fuel and raw materials are so scanty, where capital is so high (the interest rates in 1926 ranged above 7 per cent), and where labor is so cheap and so versatile, it might to many have seemed both natural and desirable if Japan could have continued indefinitely a country devoted to the small-scale production of articles for domestic consumption and of novelties and artistic objects demanded by foreigners already surfeited with machine-made goods. It is not at all certain that an "industrial revolution" is always and everywhere a blessing.

The author might well make even more emphatic than he does the fact that the rapid growth of population, which dates especially from the victory over China in 1895, is the root difficulty with labor and other conditions in Japan; and that this is the outcome of a low standard of culture and of an endless chain of poverty, ignorance and improvidence, especially among a landless peasant class. The birth-rate in the country was 33.8 per thousand of population in 1924 and 34.92 in 1925. And whatever may have been the prevailing opinion when the British Empire was in the making, the conviction appears to be growing, along with international law and the League of Nations, that the place to settle the population problem is at home. Dr. Harada agrees that emigration offers little relief, although he cannot resist the temptation to criticize the attitude of the United States toward Japanese immigration. He is less clear respecting the perils attending Japanese aspirations and efforts on the continent of Asia. The truth is that the world is already so fully occupied by peoples who wish to stay where they are that the chances for peaceful penetration as an outlet for surplus population are negligible, and that, if weaker peoples are unable to defend themselves, they will find formidable allies in other nations, whether these be actuated by jealousy or by benevolence, or both. That leaves only the solution of cutting down the birth-rate; and for this, whether it is to be accomplished by the artificial methods comprehended under the term "birth-control" or through true moral restraint, there must be offered an incentive in the shape of ambition and a prospect in life

for the average man and woman, especially the latter. We could wish, therefore, that the author had included in his rather wide peregrinations some attention to the educational system of Japan and the extent to which it promises to raise the standards and to exert a restraining hand upon this teeming population.

Quite aside from some twists in English which make it rather slow reading in places, the book is marred by a good many questionable statements and some inconsistencies. Only a few of these may be noted in approximately the order in which they occur. We are left quite in the air as between two widely different estimates as to the extent of Japanese foreign investments (pages 52–53). The expression "capital necessary to maintain the standard of living of a laborer" (page 54) suggests some reversion to the wages-fund doctrine, but proves only to mean the amount of capital invested per laborer. Mass production in the United States is attributed (page 63) to the abundant supply of raw materials and power, whereas it has probably been due much more to the call of the market, which the writer refers to only incidentally, and to the scarcity of labor. Figures as to the amount of arable land in Japan are given differently on succeeding pages (pages 78, 79). When the yield on the investment in farm land in the United States is probably not much over 4 per cent, there might be some doubt about calling the 6 per cent received in Japan "low" (page 85). In saying that "the desire to have many children is further strengthened because of the great infant mortality in Japan" (page 96), the author neglects anywhere to explain the relationship of cause and effect which the high birth-rate bears to the death-rate. Some discrepancy appears in the figures given in two different places as to the number of industrial workers (pages 98, 111). The table opposite page 138, purporting to give daily incomes in sen with several columns headed "average," is quite baffling to the uninitiated. The statement that "there is no opposition yet among the rank and file of workers directed against their leaders who have come out of universities" (page 189) is not borne out by later information which shows that these "intellectuals" are many of them radical theorists and that there have been several revolts against radicalism (pages 192, 196, 217). Little but the impression of "confusion worse confounded" could be gained from the account of the labor movement, both industrial and political, given in Chapters X and XI; but this is probably not the fault of the author, but a true picture of those factional squabbles, splits, and disruptions not unknown in the early labor movements of other countries. The author is, however, probably disposed to attach too much importance and too great influence to such organizations which at best include only a mere fraction (perhaps 6 per cent) of the industrial workers.

Yet, with all its faults, the book brings to American readers for the first time much valuable information about what is going on in industrial Japan and the reflections of an educated Japanese upon them.

WARREN B. CATLIN

Bowdoin College

Employee Stock Purchase Plans in the United States. New York: National Industrial Conference Board, Inc. 1928. xi, 245 pp.

In its study of employee stock purchase plans, the National Industrial Conference Board has contributed a very practical and comprehensive exposition of

one phase of the movement toward "labor capitalism" which Professor T. N. Carver heralded some years ago as the new economic revolution in America. As has come to be expected of Conference Board publications, the study is based on exhaustive investigation and presents its findings in an unusually well organized and readable form.

The chief merit of the Conference Board study lies in its descriptive analysis of the features of existing employee stock purchase plans. Every possible variation in the organization and operation of plans is discussed with a wealth of illustration drawn from the experience of companies reporting to the Board. Where possible, a statistical presentation of the extent of use of various features is included. In addition, reprints of typical plans and a list of all existing plans are appended. A company executive contemplating the inauguration or revision of a plan can turn here for a summary of what has been done in regard to practically every problem which may arise in employee stock purchase arrangements.

Another valuable contribution of the report is the statistical summary presented of the extent of the employee stock ownership movement. The conclusion reached is that in 1927 over a billion dollars of the stock of three hundred fifteen companies in the United States was owned by employees numbering more than eight hundred thousand. These figures are believed by the author to underestimate the full extent of employee ownership. While it is repeatedly stated that a large part of this stock is probably held by employees of supervisory or executive rank, it would be interesting if a statistical analysis of this situation had been possible.

The Board's study also includes an historical introduction concerning the employee stock ownership movement, and three concluding chapters on the effect of the new development on the participating companies, the interest of the employee in stock purchase, and the broader aspects of the movement. While a vein of optimism runs through these chapters, there is repeatedly a frank introduction of unfavorable experience or admissions of uncertainty which leave the reader with an after-taste of doubt as to the extent to which employee stock ownership is an effective solution of industrial relations problems. This doubt becomes strongest in reading the discussion of the effectiveness of employee stock ownership in solving management problems of turnover and employee interest, in solving the employee's problem of thrift, and in offering a way out of industrial conflict through the merging of labor and capital. The conclusions of the report, though by no means discouraging to those interested in the movement, but mildly reinforce Professor Carver's enthusiastic prophecies.

It is, perhaps, in dealing with the workers' attitude toward employee stock ownership that the Board's report is least convincing. It is stated that "the opposition of organized labor to employee stock ownership is easily explained when it is remembered that the American labor movement, as at present motivated, would lose its cause entirely if all the workers became owners and all the owners were workers" (page 165). This explanation presumes an acceptance by organized labor of a future for the movement which even enthusiastic employers may doubt. The belief by many workers, organized and unorganized, that employee stock ownership plans involve a partial substitution for wages, a restriction on free movement, and a concentration of savings risk is not discussed

as fully at this point as might be expected. While these problems are discussed elsewhere from the point of view of the employor or the public, the importance of the worker's attitude in an industrial relations project is, perhaps, underestimated.

The statistics in the Conference Board's study are commendable. To maintain simplicity in presentation, the classification of data in each tabulation is limited so that the reader can easily comprehend the predominating experience. In some tables it would be of interest, however, to have a classification of plans according to the size of company in relation to the particular feature or experience analyzed. Instead, the classification of plans in such tables has usually been limited to that of "active" and "inactive," a condition often dependent on causes other than degree of success.

The author in charge of the preparation of the Conference Board study, Miss Margaret L. Stecker, has organized and presented in excellent form a very valuable guide for company executives and, for the general reader, a most instructive discussion of the extent and nature of the employee stock ownership movement in this country.

J. Douglas Brown

Princeton University

This Economic World and How It May Be Improved, by Thomas Nixon Carver and Hugh W. Lester. Chicago: A. W. Shaw Company. 432 pp.

American Prosperity. Its Causes and Consequences, by Paul M. Mazur. New York: The Viking Press. 268 pp.

Professor Carver and Mr. Lester are satisfied with the world they live in. If, as they reluctantly concede, it is still only "somehow good," even that is a cause for rejoicing for it leaves a goal to attain.

Accountable for the present satisfactory state of affairs is a group of causes classified under the heading of "balancing the economic functions." Included are: (1) "A democratic tradition under which (a) every person . . . is encouraged to make the most of himself and to climb as high on the economic ladder as his ability and training will permit, and (b) all useful occupations are regarded as equally honorable"; (2) "habits of hard and prolonged work on the part of prosperous men"; (3) "an efficient system of free and universal education" which permits "thinning out the numbers in the lower and less well paid occupations"; (4) "an effective restriction of immigration"; (5) "a high standard of living on the part of the laboring classes which will lead them to postpone marriage and the raising of families until they are economically able to support them on the high standard"; (6) "widespread habits of thrift"; (7) "control of drunkenness and other forms of intemperance which destroy the necessary virtue of dependability."

The explanation of the "amazing prosperity of the people of the United States" is that in this country has been reached the finest balance of economic functions. Here equality and justice are best insured. Our system of education best fits men to reach the highest rung of the economic ladder which their abilities per-

mit them to occupy, thereby thinning the ranks of poorly paid occupations and tending to diffuse prosperity equally. Our high standard of living protects us against over-population, and our immigration laws protect our standard of living. Our successful business men do not retire to waste their time and money, leaving second-raters the task of carrying on our enterprises. And, besides, we have prohibition to thank.

To the authors there is, however, an even more basic explanation of our prosperity—to wit:

> A somewhat less extreme but equally mistaken formula for our prosperity is that we are pursuing materialistic ideals and losing our souls in the process. Materialistic ideals, however, seldom bring such prosperity as this. A somewhat sounder thesis would be that we are prospering precisely because our ideals are not materialistic, that all these things are being added unto us because we are seeking first the sound principles of justice and the sound ideals of individual behavior which are of the very essence of the Kingdom of God, and that no nation could help prospering if it pursued these principles and ideals whole-heartedly.

Throughout the book, opinions are stated as though they were acknowledged facts. The statistical data are of the type of unrefined arithmetical arrays. Anticipated objections of critics are brushed aside cavalierly. Throughout, are expressions of the authors' violent likes and dislikes—scorn of socialism, hate of Soviet Russia, devotion to the Eighteenth Amendment, and worship of our economic system. "The worst that can be said against the system is that it has not equalized prosperity or abolished the poverty of a submerged element, larger or smaller, according as the system is retarded or advanced in its development." We must interfere as little as possible with its spontaneous workings. Down with "authoritarianism!"

Mr. Mazur's discussion of American prosperity is at least an attempt to analyze an existing situation realistically. The author's critique is admittedly non-statistical. It does not prove that America is prosperous. Its assumption is "that the total income of the nation is greater than it has been in any previous period." On this simple definition of prosperity the analysis is based. Consequently the book suffers in respect to scope. Little attention is paid to such matters as standards of living, diffusion of income and inequalities of purchasing power. But within its self-imposed limitations the argument moves with precision. The reader is not led into uncharted realms of psycho-metaphysical speculation.

Mr. Mazur's thesis, in its simplest terms, is that American business is prosperous because it has acquired the technique of large-scale, economical production, on the one hand, and the art of salesmanship, on the other. Between production and distribution, the factor of primary import is the latter. Mr. Mazur is confident that our machinery of production can be geared to sustain an ever increasing prosperity. The problem is to widen the market for the output of industry. Therefore he calls this "the age of merchandising," reminding us, however, that there is a "vital balance which must be created and maintained between the needs of production and the opportunity for and cost of sales."

A widening market means not so much geographical extension of the market

place as multiplication of demand within a given area. Mr. Mazur recognizes the importance of export trade and speculates upon the effects of a reversal in our trade balance which, he predicts, will occur when "the annual payment of interest on American foreign loans will surpass the yearly investment of new capital" (*i. e.*, by Americans in foreign countries). "The results of a loss of our favorable balance of trade constitute the greatest single threat to American business which it is possible to foretell for the near future." He assures us, however, that ". . . so long as the consumption powers of America can be extended, American industry is safe."

To insure prosperity we must see to it that consumption demands constantly mount. Man's desires must and "can be developed so that they will greatly overshadow his needs." This, to Mr. Mazur, seems largely a matter of educating the consumer, through advertising, to increase his desires. Through advertising the consumer can even be taught "to want new things even before the old have been entirely consumed . . ." The boon of "obsolescence" to the manufacturer and distributor is constantly emphasized. Of course the principle of obsolescence creates problems. It makes it particularly dangerous for manufacturer, wholesaler and retailer to carry large inventories; it requires expensive national advertising campaigns; it calls for keen and far-sighted operations. Instalment selling has also "played a stellar rôle in the drama of business."

In the author's formula for prosperity, the consumer is the most important element. Hence, "the battle for consumer's loyalty" will be waged merrily, and it will be a battle of "industrial giants." We are to expect an era of consolidations such as will make the events of recent years seem insignificant.

In evaluating Mr. Mazur's contribution, we must consider it from two viewpoints. As an analysis of business structure, business methods and the problems and tendencies of manufacturing and commercial enterprises, it sets a high standard. But from the point of view of some of the assumptions upon which the exposition rests, there is room for divergent opinions. Not all will agree, for example, that waste inherent in obsolescence can form a stable foundation for expansion, or that the vast sums spent on advertising are altogether productive of welfare, or that instalment purchasing may not bring a day of reckoning. One wonders, furthermore, how Mr. Mazur would fill the consumer's pocketbook so that it will always be ready to respond. One might even be somewhat skeptical about the social consequences of unrestricted and uncontrolled consolidations.

ALFRED L. BERNHEIM

The Labor Bureau, Inc.

The Reserve Banks and the Money Market, by W. Randolph Burgess. New York: Harper and Brothers. 1927. 328 pp.

At different periods in the relatively brief history of the Federal Reserve System the public interest has been focused upon different aspects of Federal Reserve policy and practice. During the World War, the question of paramount importance was the extent to which the System could contribute to the winning of the War. In the period of acute depression which followed the War and post-

war inflation, the dominant issue, so far as the Federal Reserve was concerned, was the extent to which it could be held accountable for the distressing situation that existed.

Fortunately for all concerned, something akin to normal conditions has prevailed since 1922, and the Federal Reserve authorities have had the leisure and peace of mind essential to the development of definite and permanent policies. Beginning with 1923, the Reserve Board undertook to carry out a systematic plan for the stabilization of credit conditions. As a part of this program, the open market operations of all twelve Reserve Banks have been centered in the hands of a joint committee under Reserve Board supervision. The committee is instructed to have due regard for the effect of all open market transactions upon the credit situation. Since there is a rapidly growing interest in the whole problem of business stabilization, this phase of Federal Reserve activity now occupies the dominant place in current discussion of the System's operations. A book on the relationship between the Reserve Banks and the money market, therefore, is most timely.

Probably no one is better qualified to deal with this difficult problem than Dr. Burgess, for in addition to being an analyst of ability, he has for seven years been associated with the New York Federal Reserve Bank, where the open market operations of the whole system are centered.

When one first glances through this book, he is disposed to criticize the author for devoting more than a third of its contents to a general description of the structure and functions of the Reserve System. With literally hundreds of textbooks and other treatises in economics and finance, covering much the same ground, the reader will, at first, regret that the entire volume was not devoted to the timely and little understood subject indicated in the title. However, even though, like the reviewer, he has many times discussed with classes such topics as: "The Structure and Growth of the Reserve System," "Changes in Bank Reserves and their Use," "Changes in the Currency," "Improved Methods of Business Settlements," and "Bankers for the Government," once he has started to read one of these chapters in Burgess, he will probably read them all. The freshness and clarity of the discussion lend new interest to familiar facts. A generous use of diagrams and tables, together with up-to-date facts and figures, illustrative of actual Federal Reserve operations, contributes much to the unusual effectiveness of the presentation.

Following this general treatment of Federal Reserve functions, come three chapters which serve to depict the machinery of the money market, the more or less automatic mechanism through which any conscious credit policy must function. They present, respectively, the nature and functions of the New York money market, the development of the bankers' bill and the day-to-day changes in the New York money market.

With such a background established, the author proceeds to his real task, the description of Federal Reserve credit policy and the money market machinery employed in its enforcement. He states as the aim of such policy "to help bring about such an adaption of the volume of credit to the volume of business that every legitimate need for credit will be met, but that the volume of credit will not

be expanded beyond the legitimate need." The Reserve Board's thesis as expressed by the author is that in the formulation of discount rates "a wide range of complex considerations" must be taken into account, and that "in the last analysis reliance must be placed on judgment rather than formulas." The open market policy of the System has asserted itself almost wholly through the purchase and sale of government securities, but the author does not feel justified, in view of the limited period in which a definite credit policy has been in operation, in attempting to segregate these open market transactions from the other influences, in order to test their efficacy. There can be no doubt, however, that a substantial purchase or sale by the Reserve Banks does aid the System in enforcing a given discount policy. Dr. Burgess attributes the stability of prices since the post-war depression largely to the "cushion of Federal Reserve credit which has broken the impact of gold on bank credit." He rightly considers the extension of gold credits to central banks of Europe an effective means of securing stabilization at home, since no permanent stability could be hoped for here so long as stabilization had not been effected by important foreign nations.

In his last chapter, but one, the author undertakes to evaluate the "social consequences" of the Federal Reserve System. He concludes: (1) That interest rates are considerably more stable, (2) that, regardless of the section of the country in which he lives, a deserving borrower can obtain ample funds at all times and at reasonable rates, (3) that the banks can count upon more continuous employment of their funds.

A final chapter is appended which the author does not attempt to integrate closely with the rest of his discussion. He finds that few persons read the weekly statement of condition issued by the Reserve System and fewer still understand its real significance. He therefore presents a recent statement with illuminating comments.

Dr. Burgess is not critical of anything in the Federal Reserve mechanism or the way in which it is operated. He chronicles accurately what he finds, and seems to assume that "whatever is, is right." If one is looking for criticism of the policies laid down, or the way in which they are being carried out, he will have to look elsewhere.

GEORGE W. DOWRIE

Stanford University

The British Employment Exchange, by John Barton Seymour. London: P. S. King and Son, Ltd. 1928. 292 pp.

The general subject dealt with by this book, as its name suggests, is the organization and administration of the British system of employment exchanges. The material falls into five main groupings, plus a certain amount of supplementary data. One division is a description in some detail of the methods followed in carrying on the employment function of the exchanges, that is, in bringing together workers wanting jobs and employers wanting workers; a second describes the methods followed in administrating the British system of unemployment in-

surance entrusted to the exchanges from the beginning; a third sets forth the performance by the exchanges of various functions in aiding the government to carry out measures undertaken for the benefit of groups of unemployed persons, including government assistance to local relief works, aid in emigration, and government training classes; a fourth division discusses some of the economic advantages of the employment exchange; and a fifth presents a series of suggestions or recommendations for the future development and improvement of the system, including both immediate steps and those which will probably require time.

In addition, the author, believing that it would aid in interpreting the present work of the employment exchanges and in considering their adaptability for more extended activities prefaces the main divisions of his subject matter with a brief history of the system. He also devotes one chapter to a consideration of the statistical work of the exchanges—to the importance of systematic collection of information upon conditions of employment and unemployment, the work of this nature now being done, and desirable extensions of the present statistical work. While the aim in the book was thus to confine the discussion to the structure and methods of the exchange, and the author regarded the character and merits of government policies in this connection as outside the scope of his study, he found, however, that he could not entirely avoid a discussion of certain of these policies as distinct from their administration, and accordingly gives some attention to them.

One of the chief conclusions reached in the study is "that the State machine (the exchange system) has been forced by the pressure of other duties thrust upon it to sacrifice the development of its primary function of placing." It is expected, however, that "with an improvement in the industrial situation there should be a lessening of the heavy demands upon the exchange which have been necessitated by a long period of abnormal unemployment," and that "this should afford an opportunity for concentrating upon the chief function of the employment exchange, that of providing employers with suitable labour and employees with suitable vacancies."

The recommendations for future extension and development include, among others, the compulsory registration of vacancies by employers, an increase in the number of specialized exchanges, the development of a program for decasualizing casual labor, an increase in the number of unemployment centers, the unification of social insurance under an enlarged employment administration, and the extension of unemployment insurance to a larger proportion of the work-people.

The book brings forward valuable data for that gradual, leisurely process of testing which needs to be applied from time to time to this or any other large-scale venture in reducing unemployment and improving industrial relations and efficiency. While many readers will doubtless disagree with certain of the conclusions and recommendations, the book is a welcome and useful contribution not only to available technical material of service to workers in the exchange but also to students and to others specially interested in the subject or charged with responsibility for its development and improvement.

SHELBY M. HARRISON

Russell Sage Foundation

be expanded beyond the legitimate need." The Reserve Board's thesis as expressed by the author is that in the formulation of discount rates "a wide range of complex considerations" must be taken into account, and that "in the last analysis reliance must be placed on judgment rather than formulas." The open market policy of the System has asserted itself almost wholly through the purchase and sale of government securities, but the author does not feel justified, in view of the limited period in which a definite credit policy has been in operation, in attempting to segregate these open market transactions from the other influences, in order to test their efficacy. There can be no doubt, however, that a substantial purchase or sale by the Reserve Banks does aid the System in enforcing a given discount policy. Dr. Burgess attributes the stability of prices since the post-war depression largely to the "cushion of Federal Reserve credit which has broken the impact of gold on bank credit." He rightly considers the extension of gold credits to central banks of Europe an effective means of securing stabilization at home, since no permanent stability could be hoped for here so long as stabilization had not been effected by important foreign nations.

In his last chapter, but one, the author undertakes to evaluate the "social consequences" of the Federal Reserve System. He concludes: (1) That interest rates are considerably more stable, (2) that, regardless of the section of the country in which he lives, a deserving borrower can obtain ample funds at all times and at reasonable rates, (3) that the banks can count upon more continuous employment of their funds.

A final chapter is appended which the author does not attempt to integrate closely with the rest of his discussion. He finds that few persons read the weekly statement of condition issued by the Reserve System and fewer still understand its real significance. He therefore presents a recent statement with illuminating comments.

Dr. Burgess is not critical of anything in the Federal Reserve mechanism or the way in which it is operated. He chronicles accurately what he finds, and seems to assume that "whatever is, is right." If one is looking for criticism of the policies laid down, or the way in which they are being carried out, he will have to look elsewhere.

GEORGE W. DOWRIE

Stanford University

The British Employment Exchange, by John Barton Seymour. London: P. S. King and Son, Ltd. 1928. 292 pp.

The general subject dealt with by this book, as its name suggests, is the organization and administration of the British system of employment exchanges. The material falls into five main groupings, plus a certain amount of supplementary data. One division is a description in some detail of the methods followed in carrying on the employment function of the exchanges, that is, in bringing together workers wanting jobs and employers wanting workers; a second describes the methods followed in administrating the British system of unemployment in-

surance entrusted to the exchanges from the beginning; a third sets forth the performance by the exchanges of various functions in aiding the government to carry out measures undertaken for the benefit of groups of unemployed persons, including government assistance to local relief works, aid in emigration, and government training classes; a fourth division discusses some of the economic advantages of the employment exchange; and a fifth presents a series of suggestions or recommendations for the future development and improvement of the system, including both immediate steps and those which will probably require time.

In addition, the author, believing that it would aid in interpreting the present work of the employment exchanges and in considering their adaptability for more extended activities prefaces the main divisions of his subject matter with a brief history of the system. He also devotes one chapter to a consideration of the statistical work of the exchanges—to the importance of systematic collection of information upon conditions of employment and unemployment, the work of this nature now being done, and desirable extensions of the present statistical work. While the aim in the book was thus to confine the discussion to the structure and methods of the exchange, and the author regarded the character and merits of government policies in this connection as outside the scope of his study, he found, however, that he could not entirely avoid a discussion of certain of these policies as distinct from their administration, and accordingly gives some attention to them.

One of the chief conclusions reached in the study is "that the State machine (the exchange system) has been forced by the pressure of other duties thrust upon it to sacrifice the development of its primary function of placing." It is expected, however, that "with an improvement in the industrial situation there should be a lessening of the heavy demands upon the exchange which have been necessitated by a long period of abnormal unemployment," and that "this should afford an opportunity for concentrating upon the chief function of the employment exchange, that of providing employers with suitable labour and employees with suitable vacancies."

The recommendations for future extension and development include, among others, the compulsory registration of vacancies by employers, an increase in the number of specialized exchanges, the development of a program for decasualizing casual labor, an increase in the number of unemployment centers, the unification of social insurance under an enlarged employment administration, and the extension of unemployment insurance to a larger proportion of the work-people.

The book brings forward valuable data for that gradual, leisurely process of testing which needs to be applied from time to time to this or any other large-scale venture in reducing unemployment and improving industrial relations and efficiency. While many readers will doubtless disagree with certain of the conclusions and recommendations, the book is a welcome and useful contribution not only to available technical material of service to workers in the exchange but also to students and to others specially interested in the subject or charged with responsibility for its development and improvement.

SHELBY M. HARRISON

Russell Sage Foundation

International Yearbook of Agricultural Statistics, by the International Institute of Agriculture, Rome, 1926–27. Annual published 1927. 580 pp.

This is a compilation of statistical information relating to agriculture for all countries that collect statistics. It is entirely statistical and tabular in form; the main features are as follows: (1) Area and population, (2) apportionment of areas, as to agricultural utilization; production and numbers of livestock, (3) area, production and yield of 34 crops, (4) numbers of livestock, (5) imports and exports of 44 agricultural commodities, (6) prices of 20 commodities in important markets, (7) ocean rates for cereals and cotton, (8) production, trade and prices of important commercial fertilizers, (9) rates of exchange at New York for currency of 11 countries given one day each week.

In addition to the tables usually found in these annuals, the 1926–27 volume includes tables on international trade in sugar, silk, and wine, and consumption of commercial fertilizer in specified countries.

It is a convenient, well-arranged reference work, with comparisons for pre-war cases in which the necessary figures are available. The scope of statistics covered by these data makes this annual unique in the field of world agricultural statistics. As much material as possible is taken from official sources and is subject to relatively few errors.

Both English and French are used for all titles, but the units of measure are largely not those commonly used in the United States, which causes some inconvenience because of the necessity of making conversions.

The statistics dealing with apportionment of land area are of considerable interest for comparing the present uses of land in different countries, but their utility is somewhat restricted because different land classifications are used in the official reports of those countries. Some difficulty is also found in classifying livestock because of different age groupings, but this is unavoidable until a uniform classification is adopted by different countries.

Valuable additions would be statistics of wool production, statistics of international trade in the various meats, and statistics of apples, citrus fruits, and nuts, although official figures regarding these items are meagre. Monthly exports or imports of the leading grains and dairy products of principal countries would also be useful. In some of the earlier volumes this information was included. More extensive lists of prices of livestock and livestock products would be helpful.

Owing to the large volume of the statistics already included, extensive enlargement of the report is difficult. As at present compiled these annuals serve the greatest need, and the contents are changed or expanded as important new data become available, to meet the demand for world agricultural statistics.

OWEN L. DAWSON

United States Bureau of Agricultural Economics

The New Social Research, by Emory S. Bogardus. Los Angeles: Jesse Ray Miller. 1926. 287 pp.

The times centering around the War of 1914–18 have produced a good many new things. We have *The New Statesman*, *The New Republic*, The New School for

Social Research, and now Professor Bogardus has given us a book entitled, *The New Social Research*. The title is intriguing. What is the New Social Research? The book is an account of the methods worked out in the course of a race relations survey of the Pacific Coast, which has been in progress during the preceding two or three years. The New Social Research is, then, the methods of the social survey, as employed in this race study. The social survey is one of the well recognized techniques of sociologists, and dates further back than Booth's *Life and Labor of the People*, and has resulted in such distinguished pieces of work as the famous Pittsburgh Survey of 1908. Wherein does this survey and the methods described here differ from other surveys and survey methods? The methods set forth here differ most from the majority of social surveys in that what is generally called the case history method is stressed more, though of course the case history method has had an extensive use, dating from biographies, social workers case records, and from the psychological studies of W. I. Thomas. (Of course, The New Social Research though employing old methods may be new in that it may do better with old methods.) Of the fourteen chapters three deal with the interview and four with case histories and the personal experience. That is, about half the chapters center around the case history. Three chapters deal with public opinion. The remainder are either introductory or deal with the organization of the work. Does this mean that the survey is to rely more and more on personal life history and the diary? Perhaps, but it can hardly be so concluded from this study. In all probability, the particular subject of race relations lends itself to the case history method more than other surveys.

What is the relation of the case history method to the statistical method? The author does not discuss this point with any fullness. Only two pages of the 287 are devoted by label to statistics. The author doesn't think of statistics as the final step of proof of a hypothesis, which is the usual idea; but they are to him steps leading to opinions and attitudes, etc.

> Social statistics are factual and require interpretation. They are valuable as "leads" to opinions, attitudes, and fundamental personal experiences. They are question marks concerning the meaning of social processes. All statistics are valuable in leading the investigators to social psychological materials and interpretations. They have proved to be not ends in themselves, but guide posts to rich veins of personal experiences that require mining and bringing to the light for examination.

To the reviewer quite the contrary is the relationship. Case histories are of great exploratory value in leading to hypotheses which can only be verified by a number of cases, that is, by the statistical method. A case history describes a unique personality in a series of more or less unique experiences. Science is the discovery of new knowledge usually of a generalized relationship. A case history contains the germs of at least several hundred hypothetical relationships, each one of which, if of interest, requires proof. The question for the user of case histories is how can any one of these many hypotheses found in a case history be proved? by what method other than the quantitative? I predict that the case history will be set more and more in schedule form, and in such a manner as to be tabulated.

WILLIAM F. OGBURN

University of Chicago

RECENT LITERATURE

STATISTICAL BOOKS

BALDERSTON, C. CONBY. *Managerial Profit Sharing.* Philadelphia: University of Pennsylvania. 1928. x, 127 pp.

BAUR, FRANZ. *Korrelationsrechnung.* Leipsig, Berlin: B. G. Teubner. 1928. 57 pp.

BENNETT, MERRILL K. *Farm Cost Studies in the United States.* Stanford University Press. Food Research Institute, Miscellaneous Publications, No. 4, June, 1928. xv, 289 pp.

BENHAM, FREDERICK C. *Prosperity of Australia.* London: P. S. King and Son. 1928. x, 276 pp.

CURTIS, ROY E. *Economics, Principles and Interpretation.* Chicago and New York: A. W. Shaw Company. 1928. xviii, 879 pp.

ELMER, MANUEL C. *Technique of Social Surveys.* Los Angeles: Jesse Ray Miller. 1928. 255 pp.

FISHER, IRVING. *The Money Illusion.* New York: Adelphi Company. 1928. xv, 245 pp.

FOOD RESEARCH INSTITUTE. *British Parcels Prices: A World Wheat Price Series.* Wheat Studies, Vol. IV, No. 8. July, 1928. Stanford University, California. 18 pp.

——. *Ex-European Trade in Wheat and Flour.* Wheat Studies, Vol. IV, No. 9. August, 1928. 50 pp.

——. *Survey of the Wheat Situation,* April to July, 1928. Wheat Studies, Vol. IV, No. 10. September, 1928. 34 pp.

FURUYA, S. Y. *Japan's Foreign Exchange and Her Balance of International Payments.* New York: Columbia University Press. Studies in History, Economics and Public Law, No. 299. 1928. 208 pp.

GEBHART, JOHN C. *Funeral Costs.* New York: G. P. Putnam and Sons. 1928. xxxii, 319 pp.

GRIMES, J. A. and W. H. CRAGNE. *Principles of Valuation.* New York: Prentice Hall. 1928. xvii, 274 pp.

HAMILTON, WALTON H. and HELEN R. WRIGHT. *A Way of Order for Bituminous Coal.* Institute of Economics Series, The Macmillan Company. 1928. xiii, 365 pp.

IHRIG, KARL. *Internationale Statistik der Genossenschaften* (with 48 tables in three languages). Berlin: Verlag von Struppe und Winckler. 1928. xxiv, 275 pp.

KOREM, C. RUFUS. *Accounting Method.* University of Chicago Press. 1928. 596 pp.

LEAGUE OF NATIONS. *Methods of Compiling Housing Statistics.* International Labour Office. Studies and Reports, Series N (Statistics), No. 13. Geneva. 1928. viii, 119 pp.

MEAD, E. S. and B. OSTRALENK. *Harvey Baum: A Study of the Agricultural Revolution.* University of Pennsylvania Press. 1928. 149 pp.

MICHELS, RUDOLF K. *Cartels, Combines and Trusts in Post-War Germany.* New York: Columbia University Press. 1928. 183 pp.

NATIONAL INDUSTRIAL CONFERENCE BOARD. *The Cost of Living in the United States, 1914–1927.* New York. 1928. xiv, 142 pp.

——. *The Work of the International Labour Office.* 1928. xii, 197 pp.

RUEFF, JACQUES. *Théorie des Phénomènes Monétaires.* Paris: Payot. 1927. 368 pp.

SCHULTZ, HENRY. *Statistical Laws of Demand and Supply with Special Application to Sugar.* University of Chicago Press. 1928. xix, 228 pp.

SECRIST, HORACE (editor). *Banking Standards under the Federal Reserve System.* Northwestern University Business Studies. Chicago and New York: A. W. Shaw Company. 1928. xxxviii, 420 pp.

SWEENEY, STEPHEN B. *The Nature and Development of Aviation Insurance.* Philadelphia: University of Pennsylvania. 1927. 158 pp.

TERRY, CHARLES E. and MILDRED PELLENS. *The Opium Problem.* Committee on Drug Addiction and the Bureau of Social Hygiene. xvi, 1042 pp.

WALKER, SYDNOR H. *Social Work and the Training of Social Workers.* University of North Carolina Press. 1928. 241 pp.

WARSHAW, H. T. *Representative Industries in the United States.* New York: Henry Holt and Company. 1928. xiii, 702 pp.

WATKINS, GORDON S. *Labor Management.* Chicago and New York: A. W. Shaw Company. 1928. xv, 726 pp.

WILLIAMS, MARSHALL H. *Investment Trusts in America.* New York: The Macmillan Company. 1928. 152 pp.

WOLFENDEN, HUGH H. *Population Statistics and their Compilation.* Actuarial Society of America. 1928. viii, 144 pp.

The Workings of the Indeterminate-Sentence Law and the Parole System in Illinois. A Report to the Chairman, Parole Board of Illinois. 1928. xiv, 277 pp.

YOUNG, ALLYN A. *An Analysis of Bank Statistics for the United States.* Cambridge: Harvard University Press. 1928. vii, 78 pp.

GOVERNMENT PUBLICATIONS

UNITED STATES.

—— Bureau of Labor Statistics.

No. 464. "Retail Prices, 1890 to 1927."

No. 465. "Beneficial Activities of American Trade-Unions."

No. 466. "Settlement for Accidents to American Seamen."

No. 478. "Proceedings of the Fifteenth Annual Meeting of the International Association of Public Employment Services."

No. 479. "Activities and Functions of a State Department of Labor."

—— Department of Agriculture.

"Commerce Yearbook, 1928." Vol. I, United States. Government Printing Office. 1928. xx, 708 pp.

—— Treasury Department.

Public Health Reports:

Vol. 43.

No. 29. "Provisional Birth, Death and Infant Mortality Figures for 1927."

No. 30. "Census of State Hospitals for Mental Diseases, 1926."

No. 36. "Typhoid Fever in the United States."

FOREIGN COUNTRIES.

Australia.

"Summary of Australian Financial Statistics for the Years 1916–17 to 1926–27." Finance Bulletin No. 18. 76 pp.

la réalité en Assurances sociales"; C. Colson: "Les lois économiques et la guerre mondiale."

MISCELLANEOUS REPRINTS AND PAMPHLETS

Age Distribution and Relative Earning Capacity of Our American Population. Metropolitan Life Insurance Company, August, 1928.

Changes in Pittsburgh's Business Conditions. Some Barometers. Bureau of Business Research, University of Pittsburgh.

Longevity of College Athletes, by Louis I. Dublin. Reprinted from *Harpers' Monthly Magazine*, July, 1928.

Measures of Business Conditions in Michigan, by O. W. Blackett. Michigan Business Studies, July, 1928.

Religion and Family Size in Tallahassee, Florida, by Roland M. Harper. Reprinted from the *Journal of Heredity*, April, 1928.

Sickness Census in Montreal, December, 1926, by L. K. Frankel and L. I. Dublin. Metropolitan Life Insurance Company.

The Sources of Public Utility Capital. University of Illinois, Bureau of Business Research Bulletin, August 28, 1928.

Standards of Departmental Performance in Department Stores 1927, by Carl N. Schmalz. Michigan Business Studies, July, 1928.

A Uniform Classification of Major Offenses. Committee on Uniform Crime Records, International Association of Chiefs of Police, June, 1928.